

Math 122B

Exam III

December 4th 2014

The University of Arizona

Name: _____

Answers without adequate justification will not receive full credit, including multiple choice. Include units with your answer when appropriate, and box all answers unless an answer line is provided. By signing below I am agreeing to abide by the University of Arizona academic integrity policies and that all work done on this test is my own.

Signature: _____

Tips for Success:

- Look through the entire test before starting to prioritize questions.
- If you get stuck on a question, move on and come back to it later.
- Do a quick reality check after each question: does my answer make sense? Did I include units? Did I show all my work?
- Read over the entire test at the end to make sure you didn't miss anything.
- For each question: take a deep breath, think slowly and deliberately at first, then work quickly once you see what to do.

Requests:

- Show all your steps.
- Please box answers when possible.

1. Compute the following indefinite integrals. Simplify your answer as much as possible and leave answers exact

(a) $\int a\sqrt{x} + \frac{b}{x} dx$

(b) $\int \cosh(x) - \cos(x) dx$

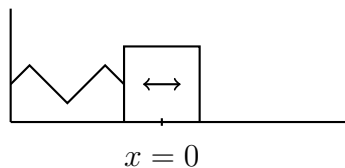
2. Compute $\int_{-1}^1 \frac{1}{1+t^2} dt$. You must show work using calculus and give an exact answer, but you can always check your result with your calculator!

3. Suppose that $h(t) = t^3$. We want to *approximate* $\int_2^5 h(t)dt$ using left-hand and right-hand Riemann sums. How many terms in our sums do we need for the error (difference between left and right sums) to be less than or equal to 0.01?

4. Compute the following limit. Hint: use L'Hôpital's rule twice.

$$\lim_{x \rightarrow 0} \frac{1 - \cos(5x)}{x^2}$$

5. A mass-spring system oscillates back and forth on a frictionless table like so:



The position in **cm** of the center of mass at time t seconds is given by the function

$$x(t) = 2 \cos(\pi t)$$

- (a) What does $\int_0^2 x'(t) dt$ represent?

- (b) Compute $\int_0^2 x'(t) dt$. Does this answer agree with your answer from part (a)?

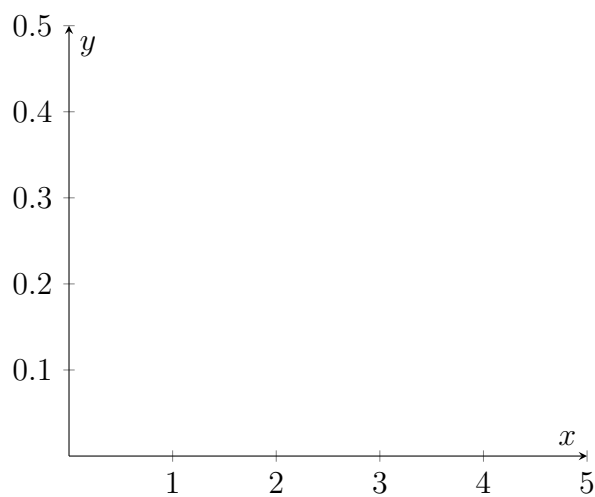
- (c) (1 point question) Write an integral expression for the *total distance traveled* by the center of mass from $t = 0$ to $t = 2$. Hint: you accumulate distance even when your velocity is negative!

6. Consider the family of functions

$$f(x) = \frac{x}{x^2 + a^2}, \quad x \geq 0, \quad a > 0$$

- (a) Find the x coordinate of the critical point of $f(x)$. Hint: remember that $x \geq 0$ and $a > 0$.

- (b) On the axes below, sketch the graphs of $y = f(x)$ for $a = 1, 2$ and 3 . You must label the critical point on each graph.



7. Suppose that $\int_{-2}^0 F(x)dx = A$ and $\int_0^2 G(x)dx = B$.

If $F(x)$ is odd and $G(x)$ is even, find $\int_{-2}^2 F(x) + 2G(x) + 1 dx$

8. Estimate $\int_{-1}^2 f(x)dx$ using the table below and a LEFT Riemann sum.

x	-1	-0.5	0	0.5	1	1.5	2
$f(x)$	2	2	3	4	6	7	8

9. Consider the differential equation

$$y'(x) = \frac{1}{x} - \frac{1}{x^2}, \quad x \geq 1$$

(a) What is the *general* solution of the equation?

(b) Find the *particular* solution of the equation so that $y(1) = -2$.

10. Let $f(x) = e^{\sin(x)}$. Find

$$\frac{d}{dx} \left[\int_0^{x^2} f(t) dt \right]$$

Hint: use one of the Fundamental Theorems of Calculus.

11. Suppose the volume of a biological cell, modeled as a sphere, is growing at a constant rate of $200\mu\text{m}^3$ per day ($1\mu\text{m} = 10^{-6}\text{m}$ is the notation for a *micrometer*). How fast is the *radius* of the cell changing when the radius is $5\mu\text{m}$? *Round your answer to three decimal places.* Recall that the volume of a sphere with radius r is given by $\frac{4}{3}\pi r^3$.

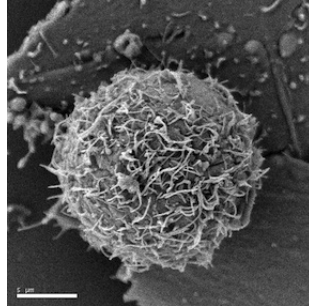


Figure 1: A (somewhat fuzzy) spherical stem cell