# Math 122B

Exam II

November 3rd 2014

The University of Arizona

Name: _____

**Answers without adequate justification will not receive full credit, including multiple choice. Include units with your answer when appropriate, and box all answers unless an answer line is provided. By signing below I am agreeing to abide by the University of Arizona academic integrity policies and that all work done on this test is my own.**

Signature: _____

**Tips for Success:**

- Look through the entire test before starting to prioritize questions.
- If you get stuck on a question, move on and come back to it later.
- Do a quick reality check after each question: does my answer make sense? Did I include units? Did I show all my work?
- Read over the entire test at the end to make sure you didn't miss anything.
- For each question: take a deep breath, think slowly and deliberately at first, then work quickly once you see what to do.

**Requests:**

- Show all your steps.
- Please box answers when possible.

1. Compute the following derivatives and simplify your answer as much as possible.

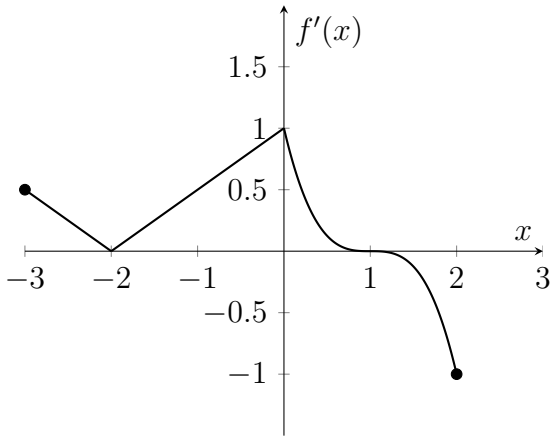
   (a) $\dfrac{d}{dt}\left[\dfrac{\sinh(at)}{e^{2t}}\right]$ ($a$ is a constant)

   (b) $\dfrac{d}{dz}\left[z\arctan(z^2)\right]$

2. Suppose that $H(x)$ is a one-to-one differentiable function such that $H(-1) = 2$ and $H'(-1) = -2$. Find the equation of the tangent line to $y = H^{-1}(x)$ at $x = 2$.

3. Find the $t$ values of the global maximum and minimum of $f(t) = \cos(2\pi t - \pi/4))$ on the interval $[0, 1)$. You must use calculus to justify your answer.

4. Consider the graph of $y = f'(x)$ below.




(a) Using the graph above, find and classify the critical points of $f(x)$. You must show work using a sign chart.

(b) Suppose that $f(-1) = 10$. Write the equation of a linear function $L(x)$ which approximates $f(x)$ near $x = -1$.

(c) Use your answer from part (b) to estimate $f(-1/2)$.

(d) Is your answer from part (c) an over or under estimate? Why?

5. Find the value of $k$ so that $f(x) = x^3 - kx^2 + kx + k$ has an inflection point at $x = 3$. You must show work using a sign chart.

6. Consider the implicit curve in the $(x, y)$ plane defined by the equation $x^2y - y^2x = 1$.

   (a) Find an equation for $\frac{dy}{dx}$ in terms of $x$ and $y$.

   (b) Find an $(x, y)$ pair with $x > 0$ and $y > 0$ where the tangent to the curve is **vertical**. Hint: you will need to use both the original equation and the equation for $\frac{dy}{dx}$.

7. A cylindrical aluminum beverage can is to be made to hold a volume of 350 cubic centimeters. It will be built from two materials: the circular top and bottom pieces cost \$0.02 per square cm, while the side piece costs \$0.01 per square cm. **Find the radius $r$ and height $h$ that minimize the total cost of building the can**.

   - Hints: write down the objective function in words first, then in terms of variables. Then use the constraint to eliminate a variable.
   - You must provide a single variable function with domain, and use calculus to find the global minimum on your domain.
   - You may round your answers to three decimal places.

8. Answer the following questions with T or F. No explanation is needed - they are worth 1 point each, all or nothing.

(a) $\frac{d}{dx}[f^{-1}(x)] = \frac{1}{f'(x)}$

(a) _____

(b) If $f(x)$ is a continuous function on the interval $(a, b)$, then $f(x)$ has a global maximum and minimum value on $(a, b)$.

(b) _____

(c) If $f(x)$ is differentiable on $(-\infty, \infty)$ and $f(0) = f(10)$, then there is a $c \in (0, 10)$ such that $f'(c) = 0$.

(c) _____

(d) $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$

(d) _____

(e) If $f(x)$ is such that $f'(1) = 0$ and $f''(1) = 0$, then $f(x)$ has neither a maximum nor a minimum at $x = 1$.

(e) _____

(f) Math 122B is the greatest class ever.

(f) _____