Math 223 – Vector Calculus Practice Exam 3

Author: Arlo Caine Name:__________________________

Directions: Read all questions carefully. Use a pencil and erase all unnecessary marks. Show all of your work in the space provided and display your answer on the line given if requested. You will lose points if you make an approximation and fail to indicate the approximation. Be careful to use proper notation to indicate vector versus scalar quantities as well.

1. Determine whether the following are true or false (T or F). You do not need to give your reasons.

   (a) ___ The parametric curve $x = t^2$, $y = 2$, $z = t$ is a parabola in space.

   (b) ___ If one particle moves with position $\vec{r}_1(t) = \cos(t)\vec{i} + \sin(t)\vec{j} + t\vec{k}$ while another is moves with position $\vec{r}_2(t) = \vec{i} + (10 - t)\vec{k}$ then the two particles will collide at some time.

   (c) ___ If a particle is moving along a parameterized curve $\vec{r}(t)$ then the acceleration vector at any point is perpendicular to the velocity at that point.

   (d) ___ If a particle moves with constant velocity then the path must be a line.

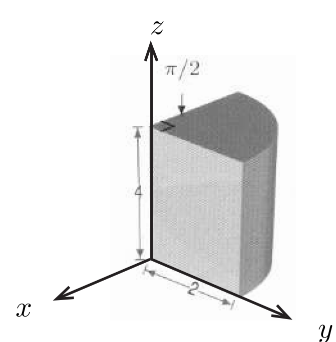
   (e) ___ There is a region R over which $\int_R f\,dA$ cannot be evaluated by a *single* iterated integral in the order $\int\int f\,dx\,dy$ or $\int\int f\,dy\,dx$.

   (f) ___ If the vector fields $\vec{F}$ and $\vec{G}$ have $\int_C \vec{F} \cdot d\vec{r} = \int_C \vec{G} \cdot d\vec{r}$ for a particular path C then $\vec{F} = \vec{G}$.

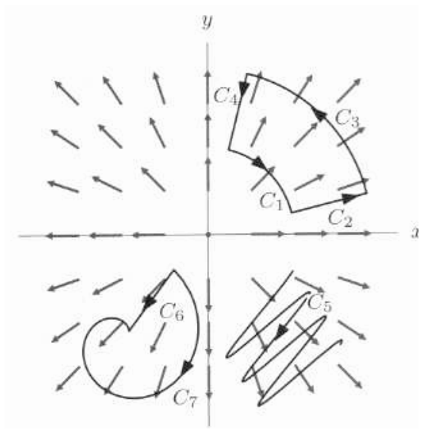
   (g) ___ If $\vec{F} = \vec{i}$ is a vector field in 2-space, then $\int_C \vec{F} \cdot d\vec{r} > 0$, where C is the oriented line from $(0, 0)$ to $(1, 0)$.

   (h) ___ If $\vec{F}$ is path-independent and defined on the whole plane, and C is any closed curve then $\int_C \vec{F} \cdot d\vec{r} = 0$.

2. Setup two integrals, one in rectangular coordinates, one in cylindrical coordinates to calculate the volume of the figure shown. Choose one of your integrals to evaluate and do so.



3. Let $\vec{F}(x, y)$ be the path-independent vector field shown in the figure. The vector field $\vec{F}$ associates to each point in the plane a unit vector pointing radially outward and thus has a singularity at the origin. The curves C_1, C_2, $\ldots$, C_7 have the direction shown. Consider the line integrals $\int_{C_i} \vec{F} \cdot d\vec{r}$ for $i = 1, 2, \ldots, 7$.



Without calculating any integrals perform the following tasks and give a one or two sentence explanation of each answer.

(a) List all the line integrals which you expect to be zero.

(b) List all the line integrals which you expect to be negative.

(c) Arrange the positive line integrals in ascending order.

4. A particle, following a straight line constant speed path through space, passes "downward" through a permeable barrier which is in the shape of the graph of $z = 10 - x^2 - y^2$, where x, y and z are measured in meters. It passes through the point $(2, 2, 2)$ in a direction perpendicular to the barrier with speed $\sqrt{33}$ m/s at time $t = 0$. Does it strike the barrier again? If your answer is "no", explain why. If your answer is "yes" find the time of intersection.

5. The table shows some of the values of a function $f(x, y)$.

$x \backslash y$	3	6	9	12
-4	10	12	15	19
-2	8	10	12	15
0	6	7	9	11
2	4	5	6	7

(a) Find parametric equations for the upper semi-circle of radius 3 traversed from $(2, 6)$ to $(-4, 6)$.

(b) Let C denote the curve in part (a). Estimate $\int_C \nabla f \cdot d\vec{r}$.

6. Sketch the region R in the plane whose boundary is the oriented curve $C = C_1 + C_2 + C_3$ where

$$C_1 : \begin{cases} x &= t \\ y &= \sin(t) \end{cases} \quad \text{and } C_2 : \begin{cases} x &= 2\pi - \sin(t) \\ y &= t \end{cases} \quad \text{and } C_3 : \begin{cases} x &= 2\pi - t \\ y &= 2\pi - t \end{cases}$$

where $0 \leq t \leq 2\pi$ in each parameterization. Use Green's Theorem to compute the area of the region R.