Math 223 – Vector Calculus Practice Exam 2

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Directions: Read all questions carefully. Use a pencil and erase all unnecessary marks. Show all of your work in the space provided and display your answer on the line given if requested. You will lose points if you make an approximation and fail to indicate the approximation. Be careful to use proper notation to indicate vector versus scalar quantities as well.

1. Determine whether the following are true or false (T or F). You do not need to give your reasons.

   (a) ___ Level sets of a function $f(x, y, z)$ of different levels never intersect.

   (b) ___ If $f(x, y) = k$ for all points (x, y) in a region R then $\int\int_R f(x, y)\, dA = k \cdot \text{Area}(R)$.

   (c) ___ If f and g are two functions continuous on a region R then $\int\int_R fg\, dA = \int\int_R f\, dA \int\int_R g\, dA$.

   (d) ___ If W_1 and W_2 are solid regions of space with volume$(W_1) >$ volume(W_2) then

   $$\int\int\int_{W_1} f\, dV > \int\int\int_{W_2} f\, dV.$$

   (e) ___ The function $z(u, v) = u \cos v$ satisfies the equation

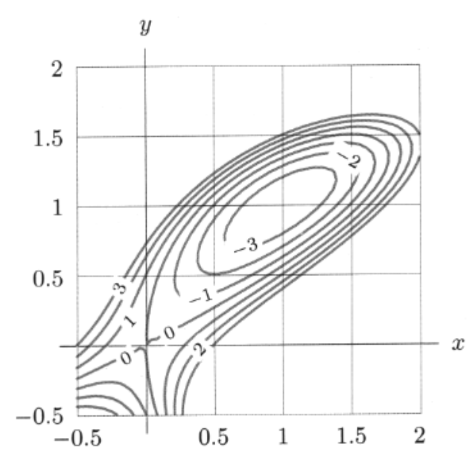
   $$\cos v \frac{\partial z}{\partial u} - \frac{\sin v}{u} \frac{\partial z}{\partial v} = 1.$$

   (f) ___ The local linearization of $f(x, y) = x^2 + y^2$ at the point $(1, 1)$ gives an overestimate of the value of $f(x, y)$ at the point $(1.1, 1.2)$.

   (g) ___ If you know each of the directional derivatives of f at (a, b, c) in the directions of 3 perpendicular unit vectors in space then you can determine the gradient of f at the point (a, b, c).

2. The figure shows the contours for a function $f(x, y)$. Using this contour diagram indicate with a check mark whether the given derivative is positive, negative, or zero. Let $\vec{u} = (1/\sqrt{2})(\vec{i} + \vec{j})$.

	positive	negative	zero
$f_x(0.5, 0)$			
$f_y(1.5, 1)$			
$f_{xx}(0.5, 0.5)$			
$f_{yy}(0.5, 0)$			
$f_{xy}(0.5, 1)$			
$f_{\vec{u}}(1, 0.5)$			



3. Let $g(x, y, z) = z - f(x, y)$ where $f(x, y) = \sqrt{1 - y^2}$.

(a) Sketch a graph of the level surface $g = 0$.

(b) Explain why the vectors $\vec{v} = \vec{i} + f_x(3, -1/\sqrt{2})\vec{k}$ and $\vec{w} = \vec{j} + f_y(3, -1/\sqrt{2})\vec{k}$ would be tangent to the level surface if their tails were attached at the point $(3, -1/\sqrt{2}, 1/\sqrt{2})$.

(c) Determine the component of $\vec{v} \times \vec{w}$ perpendicular to $\operatorname{grad} g(3, -1/\sqrt{2}, 1/\sqrt{2})$.

(d) Calculate the directional derivative $g_{\vec{u}}(3, -1/\sqrt{2}, 1/\sqrt{2})$ where $\vec{u} = \frac{1}{\sqrt{5}}\vec{i} + \frac{2}{\sqrt{5}}\vec{j}$.

(e) Find the equation of the plane tangent to the level surface $g = 0$ at the point $(3, -1/\sqrt{2}, 1/\sqrt{2})$.

4. The level surfaces $f = -1$ and $g = 0$ of the functions $f(x, y, z) = z^2 - x^2 - y^2$ and $g(x, y, z) = y^2 + z^2 - 1$ intersect along a curve that passes through the point $P = (1, 1/\sqrt{2}, 1/\sqrt{2})$. Use the gradients of f and g to find a vector which which would be tangent to this curve if its tail was attached at the point P.

5. The voltage V (in volts) across a circuit is given by Ohm's Law: $V = IR$, where I is the current (in amps) flowing through the circuit and R is the resistance (in ohms). If we place two circuits, with resistance R_1 and R_2 in parallel, then their combined resistance, R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Suppose the current is 2 amps and increasing at a rate of 10^{-2} amps/sec and R_1 is 3 ohms and decreasing at a 0.5 ohms/sec while R_2 is 5 ohms and decreasing at 0.1 ohms/sec. Calculate the rate at which the voltage is changing.

6. The hull of a boat has width $w(x, y)$ feet at a point x feet from the front and y feet below the water line. Values of w are in given in the table.

$y\backslash x$	0	10	20	30	40	50	60
0	2	8	13	16	17	16	10
2	1	4	8	10	11	10	8
4	0	3	4	6	7	6	4
6	0	1	2	3	4	3	2
8	0	0	1	1	1	1	1

(a) Set up a definite integral that calculates the volume of the hull below the water line. Explain each step.

(b) Estimate your integral using the table.

7. Sketch the region of integration and evaluate the iterated integral

$$\int_{-2}^{0} \int_{0}^{\sqrt{4-x^2}} x \, dy \, dx + \int_{0}^{1} \int_{1-x}^{2} x \, dy \, dx$$

Show each step of the calculation.