Math 223 – Vector Calculus Practice Exam 1

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Directions: Read all questions carefully. Use a pencil and erase all unnecessary marks. Show all of your work in the space provided and display your answer on the line given if requested. You will lose points if you make an approximation and fail to indicate the approximation. Be careful to use proper notation to indicate vector versus scalar quantities as well.

1. Determine whether the following are true or false (T or F). You do not need to give your reasons.

   (a) ___ Two contours of $f(x, y)$ with different heights never intersect.

   (b) ___ There is only one point in the yz-plane that is a distance 3 from the point $(3, 0, 0)$.

   (c) ___ The graphs of $f(x, y) = \sin(xy)$ and $f(x, y) = \sin(xy) + 2$ do not intersect.

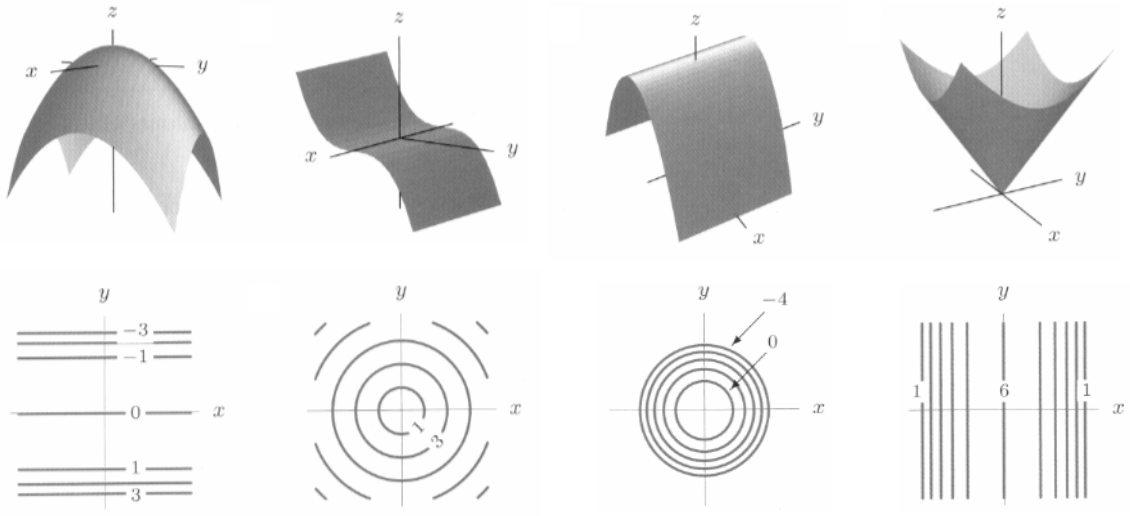
   (d) ___ There is no linear function whose graph is a plane parallel to the xz-plane.

   (e) ___ The vectors $3\vec{i} - 2\vec{j} - 4\vec{k}$ and $-39\vec{i} + 26\vec{j} + 52\vec{k}$ are parallel.

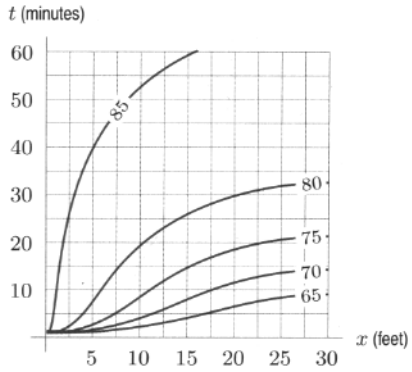
   (f) ___ The quantity $((\vec{b} \times \vec{a}) \cdot \vec{c})\vec{a}$ is a vector.

   (g) ___ If $\vec{v}$ and $\vec{w}$ are any two vectors then $\|\vec{v} + \vec{w}\| = \|\vec{v}\| + \|\vec{w}\|$.

2. Match the graph with the contour diagram.



3. You are in a room 30 feet long with a heater at one end. In the morning the room is 65°F. You turn on the heater, which quickly warms the room up to 85°F. Let $H(x,t)$ be the temperature x feet from the heater, t minutes after the heater is turned on. The figure shows the contour diagram for H.



(a) How warm is it 10 feet from the heater 5 minutes after it was turned on?

(b) How far from the heater was the temperature changing at a approximately a rate of 1°F/min, 5 minutes after the heater was turned on.

4. Sketch a picture AND describe in words the following sets of points.

(a) The set of points in space whose distance from $(1,-2,3)$ is 5.

(b) The set of points in space whose distance from the line $y=x$ in the xy-plane is the same as its distance to the plane $x+y=0$.

(c) The set of points in space which lie in the plane $z=4$ whose distance from the z-axis is 3.

(d) The set of points in the plane for which the product of the distances from the point to the points $(1,0)$ and $(-1,0)$ is one.

5. Let $P_0 = (1,0,0)$, $\vec{n}_1 = \vec{i} + \vec{j} + \vec{k}$, and $\vec{n}_2 = \vec{i} - 2\vec{j} + \vec{k}$. Let $P = (x,y,z)$ be a variable point. Write $\vec{P} = x\vec{i} + y\vec{j} + z\vec{k}$ for the position vector of that point and $\overrightarrow{P_0P}$ for the displacement vector from P_0 to P. The following equations describe a line or a plane or a point in 3-space. Explain which is which.

   (a) $\overrightarrow{P_0P} \times \vec{n}_1 = \vec{0}$

   (b) $\overrightarrow{P_0P} \cdot \vec{n}_1 = 0$

   (c) $\vec{P} \cdot \vec{n}_1 = 3$

   (d) $(\vec{P} \times \vec{i}) \cdot \vec{j} = 0$

   (e) $\begin{cases} \overrightarrow{P_0P} \cdot \vec{n}_1 &= 0 \\ \overrightarrow{P_0P} \cdot \vec{n}_2 &= 0 \end{cases}$

   (f) $\begin{cases} \overrightarrow{P_0P} \cdot \vec{n}_1 &= 0 \\ \overrightarrow{P_0P} \cdot \vec{n}_1 &= 0 \end{cases}$

6. Suppose that you are standing straight upright with your feet at the point $(-1,1,1)$. Your spine is aligned in the $\vec{j}$ direction and you are facing the $-\vec{i}$ direction. From this perspective is the point $(1,2,3)$ in front of or behind you? to your left or to your right? above or below you? Explain.

7. Find a vector parallel to the intersection of the planes described by $x + y - z = 0$ and $2x - y + z = 0$.

8. The table below gives a partially completed table of a **linear** function $f(x, y)$.

x y	1	2	3	4
1	2	5	8	11
2				
3		1	4	
4	-4			5

(a) Complete the table using the fact that f is linear.

(b) Find c, m, and n such that the data is modelled by $f(x, y) = c + mx + ny$.

9. A pilot is flying a small airplane with an airspeed of 100 nm/hr. The vertical speed indicator reads that she is climbing at a rate of 500 ft/min. She is flying a heading chosen so that with the wind blowing southeast at 25 nm/hr she will follow a course due east. To do the following problem you may find it useful to know that there are 6080 feet in a nautical mile (nm).

(a) Compute the direction of her heading and express it in terms of degrees north/south of east.

(b) Compute her ground speed.