Homework Sample

Problem: A 30 kilogram weight is hanging at the end of a 40 meter rope. The rope weighs $\frac{1}{2}$ kilogram per meter. Compute the work required to raise the rope and weight to the height of the top of the rope.

Solution: The work required to raise the weight is 30 kg $\cdot$ 40 m = 1200 joules. To compute the work required to raise the rope, divide the rope into n equal pieces of length Δh. The ith piece will then weigh $\frac{1}{2} \cdot \Delta h$ kg. If h_i is the distance from the top of the ith piece to the top, the work required to raise the ith piece is approximately $\frac{1}{2} \cdot \Delta h \cdot h_i$. Therefore, for a given n, the work required to lift the entire rope is approximately

$$\sum_{i=1}^{n} \frac{1}{2} \cdot \Delta h \cdot h_i.$$

As $n \to \infty$, the error in this approximation approaches 0, and

$$\sum_{i=1}^{n} \frac{1}{2} \cdot \Delta h \cdot h_i \to \int_0^{40} \frac{1}{2} h \, dh = \frac{1}{2} \left[\frac{h^2}{2} \right]_0^{40} = \frac{1}{4}(1600 - 0) = 400.$$

Therefore, the total work required to lift the rope and weight is 1200+400=1600 joules.