Math 223 - Section 010
Vector Fields Lab with Solutions

In this lab, you will use the applet at http://www.falstad.com/vector3d/ to develop some intuition about vector fields in space. Some general comments about the applet:

- The "Particles (Vel.)" and "Streamlines" display modes (in the second drop-down menu from the top) are typically the most useful.
- In the "Field Vectors" display mode, the magnitude of each vector is indicated by its color, rather than by the length of the arrow.
- Many of the predefined fields in the "Field selection" menu have been scaled by constant factors. You can scale the vector fields you enter either by multiplying by a scalar in each formula or by adjusting the "Field Strength" slider.
- Click the "Reset" button to start more particles flying around the field.
- To enter a field, choose "user-defined field" from the "Field selection" drop-down menu. Three boxes will appear underneath the menus. The first box represents the $\vec{i}$ component of the field, the second box represents the $\vec{j}$ component, and the third box represents the $\vec{k}$ component.
- If your vector field is not defined at some points (e.g., $\vec{F} = \frac{1}{z}$ is not defined on the xy-plane) the applet may show you something strange at those points.

1. Find formulas for the following vector fields from the first drop-down menu under "Field selection."

   a. One direction
   Solution: $\vec{i}$

   b. Linear radial
   Solution: $-x\vec{i} - y\vec{j} - z\vec{k}$

   c. Constant radial
   Solution: $\frac{-x\vec{i}-y\vec{j}-z\vec{k}}{\sqrt{x^2+y^2+z^2}}$

   d. Linear to xy-plane
   Solution: $-z\vec{k}$

   e. Constant to xy plane
   Solution: $-\frac{z}{|z|}\vec{k}$

f. Linear rotational
Solution: $-y\vec{i} + x\vec{j}$

g. Constant rotational
Solution: $\frac{-y\vec{i}+x\vec{j}}{\sqrt{x^2+y^2}}$

h. Helical
Solution: $-y\vec{i} + x\vec{j} + \vec{k}$

2. Give a formula for a vector field that causes a particle at any point in space to rise straight up (in the direction of the vector $\vec{k}$) at increasing speed.
Solution: $e^z\vec{k}$

3. Give a formula for a vector field that causes a particle at any point in space to move in the direction of the vector $\vec{i} + \vec{j} + \vec{k}$ at increasing speed.
Solution: $e^{x+y+z}(\vec{i} + \vec{j} + \vec{k})$

4. Find a vector field that causes particles to spiral up toward the xy-plane from below and spiral down toward the xy-plane from above. Design your field so that particles spiral in the same direction above and below the xy-plane.
Solution: $-y\vec{i} + x\vec{j} - z\vec{k}$

5. Find a vector field that causes particles to spiral upward in such a way that

   a. particles below the xy-plane spiral counterclockwise, and particles above the xy-plane spiral clockwise (when viewed from above).
   Solution: $z(y\vec{i} - x\vec{j}) + \vec{k}$

   b. any particle that starts below the xy-plane never ranges above the xy-plane.
   Solution: $z(y\vec{i} - x\vec{j}) + |z|\vec{k}$

6. (Bonus) Give a formula for a vector field that causes particles to revolve about the line $z = y$, $x = 0$.
Solution: $(y - z)\vec{i} - x\vec{j} + x\vec{k}$