Math 215 - Section 004
Hints for Section 3.6

- **30.** The maps T and S are both maps from $\mathbb{R}^2$ to $\mathbb{R}^2$, so the matrices $[T]$ and $[S]$ are both 2×2. For example, the matrix for S is $[S] = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$. To find a matrix for $[S \circ T]$, use the fact that the formula for $S \circ T$ is

$$(S \circ T)(x_1, x_2) = S(T(x_1, x_2)) = S(x_1 - x_2, x_1 - x_2) = (2(x_1 - x_2), -(x_1 - x_2)).$$

- **43.** You may assume that reflection over a line is linear (this is obvious from the geometry). This makes it easy to write down formulas for F_n, F_m, and F_l: just see what they do to $[0, 1]$ and $[1, 0]$ and put those results into the columns of a matrix. It will be helpful to make the following definitions. Let F_n be the reflection across the line that makes an angle of α (degrees or radians) with the positive x-axis. Similarly, let β and γ be the angles of the lines for F_m and F_l.

- **44.** Let $\vec{v}(t) = [x(t), y(t)] = [a + bt, c + dt]$ be the equation for a line. Now let T be a linear map from $\mathbb{R}^2$ to $\mathbb{R}^2$ with matrix $[T] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. Now multiply $[T]$ by $\vec{v}(t)$ and see what you get.

- **54.** Write down the formula for $[T]$. In terms of this formula, write down a formula for $T(\vec{x}) = [T]\vec{x}$, where $\vec{x} = [x_1, \ldots, x_n]$. The result you are asked to prove will be obvious.