

Comments on Problem 42 from Section 1.3

Problem: Prove that the distance between two parallel planes, \mathcal{A}_1 and \mathcal{A}_2 , with equations $\vec{n} \cdot \vec{x} = d_1$ and $\vec{n} \cdot \vec{x} = d_2$, respectively, is $\frac{|d_1 - d_2|}{\|\vec{n}\|}$.

It is fairly easy to convince yourself that this is true by drawing a picture and computing a projection. However, it would be best to give a proof that does not rely on a picture. The following is an outline of one possible proof:

1. Choose a point P on \mathcal{A}_1 , and parameterize a line $\vec{x}(t)$ which starts at P and moves in the direction of the normal vector \vec{n} .
2. Show that this line intersects the plane \mathcal{A}_2 . In particular, find an explicit expression for the value of t for which $\vec{x}(t)$ is on \mathcal{A}_2 . Call this value t_0 , and let $R = \vec{x}(t_0)$.
3. Use the Pythagorean Theorem to show that if Q is any other point on \mathcal{A}_2 , then the length of \overrightarrow{PR} is less than the length of \overrightarrow{PQ} .
4. Use the explicit expression you found for t_0 to compute the length of \overrightarrow{PR} .
5. Argue that since this length does not depend on the choice of P , $\|\overrightarrow{PR}\|$ is the distance between the planes \mathcal{A}_1 and \mathcal{A}_2 .

A reasonable question to ask at this point is whether this theorem (or something very similar) makes sense in dimensions higher than 3. If so, does the proof outlined above work in the higher dimensional cases?