Math 215 - Section 004

Final Exam Review

1. Let $\vec{u} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 6 \\ -1 \\ -1 \end{bmatrix}$. Give a decimal approximation for the angle between $\vec{u}$ and $\vec{v}$.

2. Find the distance between the parallel lines

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} -2 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} + t \begin{bmatrix} 4 \\ -2 \end{bmatrix}.$$

3. Use Gauss-Jordan elimination to solve the following linear system. Show all of your steps.

$$\begin{aligned} x_1 + 2x_2 - 4x_3 - 4x_4 + 5x_5 &= 3 \\ 2x_1 + 4x_2 + 2x_5 &= 2 \\ 2x_1 + 3x_2 + 2x_3 + x_4 + 5x_5 &= 4 \\ -x_1 + x_2 + 3x_3 + 6x_4 + 5x_5 &= -1 \end{aligned}$$

4. Show that a linear system $A\vec{x} = \vec{b}$ is consistent if and only if $\vec{b}$ is a linear combination of the columns of A.

5. TRUE or FALSE: Every diagonalizable matrix has at least one eigenvalue.

6. Give an example of a 5×5 matrix that is invertible and not diagonalizable or explain why no such matrix exists.

7. Give an example of a 5×5 matrix that is diagonalizable and not invertible or explain why no such matrix exists.

8. Give an example of a pair of nonzero matrices A and B whose product is the zero matrix. Notice that this cannot happen in the 1×1 case. Can you find **invertible** matrices whose product is 0?

9. Find a pair of invertible, 2×2 matrices that are not row-equivalent or explain why no such pair exists.

10. Assuming that $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -2$, find $\begin{vmatrix} a & b & 5c-2a \\ d & e & 5f-2d \\ g & h & 5i-2g \end{vmatrix}$.

11. Consider the sets of vectors $[x, y, z]$ in $\mathbb{R}^3$ satisfying the following conditions:

12. a. $y^2 + z^2 = 0$
    b. $(x-z)^{100} + (x+z)^{100} = 0$
    c. $x + 2y = z$
    d. $xy + yz = 0$
    e. z is a rational number

    Which of the above conditions defines a subspace of $\mathbb{R}^3$? (Be careful. Some of these are tricky.)

13. Find two 4×4 matrices A and B which have the same eigenvalues but different eigenvectors. (Hint: Similar matrices have the same eigenvalues.)

14. Let A and B be $n \times n$ matrices. Suppose B is not invertible.

    a. Explain in geometric terms why AB cannot be invertible.

    b. Use the determinant to prove that AB is not invertible.

15. A 23x23 matrix has three eigenvalues λ_1, λ_2, and λ_3. Suppose that the dimension of E_{λ_1} is 5 and the dimension of E_{λ_2} is 7. What must the dimension of E_{λ_3} be if the matrix is diagonalizable? Explain your answer.

16. Suppose A is an invertible matrix. Prove that if A diagonalizable, then A^{-1} is diagonalizable.