Math 129 - Section 001
Quiz on 11.6 with Answers

Write clearly and ***show all of your work.*** *Good luck.*

1. Water leaks out of a barrel at a rate proportional to the square root of the depth of the water at that time. If the water level starts at 30 inches and drops to 28 inches in 1 hour, how long will it take for all of the water to leak out of the barrel?

   Answer: Solve the differential equation $\frac{dy}{dt} = -k\sqrt{y}$ to get $2\sqrt{y} = -kt + C$. Then

   $$y(0) = 30 \Rightarrow C = \sqrt{120},$$

   and

   $$y(1) = 28 \Rightarrow k = \sqrt{120} - \sqrt{112}.$$

   So $2\sqrt{y} = \sqrt{120} - (\sqrt{120} - \sqrt{112})t$. Since $y = 0 \Leftrightarrow \sqrt{y} = 0$, set $\sqrt{120} - (\sqrt{120} - \sqrt{112})t = 0$ and solve to get $t = \frac{\sqrt{120}}{\sqrt{120} - \sqrt{112}} \approx 29.5$ hours.

2. When a gas expands without gain or loss of heat, the rate of change of pressure with respect to volume is proportional to pressure divided by volume. Find a law relating pressure and volume in this case.

   Answer: Solve the differential equation $\frac{dP}{dV} = k\frac{P}{V}$. The general solution is $P = AV^k$.