Math 124 - Section 012
Quiz on 3.5 (with solutions)

*Write clearly and **show all of your work.** Good luck.*

1. Compute $\frac{d}{dy}\left(\sqrt{\frac{1-\cos^2 y}{1-\sin^2 y}}\right)$.

   Solution: $\frac{d}{dy}\left(\sqrt{\frac{1-\cos^2 y}{1-\sin^2 y}}\right) = \frac{d}{dy}\left(\sqrt{\frac{\sin^2 y}{\cos^2 y}}\right) = \frac{d}{dy}\left(\sqrt{\tan^2 y}\right) = \frac{d}{dy}\left(\sqrt{\tan^2 y}\right)$

   $= \frac{d}{dy}\left(|\tan y|\right) = \begin{cases} \frac{1}{\cos^2 y} & \text{if } \tan y > 0 \\ -\frac{1}{\cos^2 y} & \text{if } \tan y < 0 \\ \text{undefined} & \text{if } \tan y = 0 \end{cases}$

2. (10 points) Suppose an object of mass m is oscillating at the end of a spring, and that $T = A\sin\left(\left(\sqrt{\frac{k}{m}}\right)t\right)$ represents the distance of the object from its equilibrium position at time t. (A and k are constants.)

   a. Find a time at which the mass is moving fastest.

   Solution: $\frac{dT}{dt} = A\cos\left(\sqrt{\frac{k}{m}}t\right)\cdot\sqrt{\frac{k}{m}}$ attains its maximum at $t = 0$, $\frac{2\pi}{\sqrt{k/m}}$, $\frac{4\pi}{\sqrt{k/m}}$, etc.

   b. Find a time at which the mass is accelerating fastest.

   Solution: $\frac{d^2T}{dt^2} = -A\sin\left(\sqrt{\frac{k}{m}}t\right)\cdot\frac{k}{m}$ attains its maximum at $t = \frac{3}{4}\cdot\frac{2\pi}{\sqrt{k/m}}$, $t = \frac{3}{4}\cdot\frac{2\pi}{\sqrt{k/m}} + \frac{2\pi}{\sqrt{k/m}}$, etc.