Math 124 - Section 012
Quiz on 1.8

Write clearly and ***show all of your work.*** *Good luck.*

1. Assuming that limits as $x \to \infty$ have the properties listed in section 1.8 for limits as $x \to c$, use algebraic manipulations to evaluate $\lim\limits_{x\to\infty} \dfrac{5x^{\left(\frac{3}{2}\right)}+1}{3x+4x^{\left(\frac{3}{2}\right)}+2}$.

   Solution:

$$\lim_{x\to\infty} \frac{5x^{\left(\frac{3}{2}\right)}+1}{3x+4x^{\left(\frac{3}{2}\right)}+2} = \lim_{x\to\infty} \frac{5x^{\left(\frac{3}{2}\right)}+1}{3x+4x^{\left(\frac{3}{2}\right)}+2}\left(\frac{x^{-\frac{3}{2}}}{x^{-\frac{3}{2}}}\right) \tag{1}$$

$$= \lim_{x\to\infty} \frac{5+x^{\left(-\frac{3}{2}\right)}}{3x^{\left(-\frac{1}{2}\right)}+4+2x^{\left(-\frac{3}{2}\right)}} = \frac{\lim\limits_{x\to\infty} 5+\lim\limits_{x\to\infty} x^{\left(-\frac{3}{2}\right)}}{\lim\limits_{x\to\infty} 3x^{\left(-\frac{1}{2}\right)}+\lim\limits_{x\to\infty} 4+\lim\limits_{x\to\infty} 2x^{\left(-\frac{3}{2}\right)}} \tag{2}$$

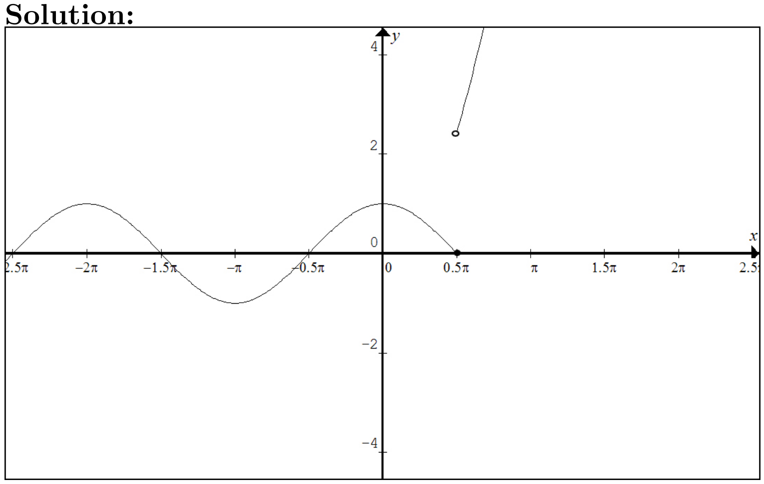
$$= \frac{\lim\limits_{x\to\infty} 5+\lim\limits_{x\to\infty} x^{\left(-\frac{3}{2}\right)}}{3\lim\limits_{x\to\infty} x^{\left(-\frac{1}{2}\right)}+\lim\limits_{x\to\infty} 4+2\lim\limits_{x\to\infty} x^{\left(-\frac{3}{2}\right)}} = \frac{5+0}{0+4+0} = \frac{5}{4} \tag{3}$$

   The claim that $\lim\limits_{x\to\infty} x^{\left(-\frac{3}{2}\right)} = \lim\limits_{x\to\infty} x^{\left(-\frac{1}{2}\right)} = 0$ in line (3) is justified by the fact that $x^{\frac{3}{2}}$ and $x^{\frac{1}{2}}$ grow without bound as $x \to \infty$.

2. Let $f(x) = \begin{cases} \cos x & \text{if } x \leq \frac{\pi}{2} \\ x^2 & \text{if } x > \frac{\pi}{2} \end{cases}$

a. Sketch the graph of $f(x)$.

Solution:



b. Compute $\lim_{x \to \frac{\pi}{2}^-} (x + f(x))$ or explain why it does not exist.

Solution: The properties for two-sided limits listed in the table on p51 of the text are also true for one-sided limits. Since this fact is not mentioned in the book, I did not grade this problem. However, you will be asked to use these properties to compute one-sided limits on the test.

From the graph, it is clear that $\lim_{x \to \frac{\pi}{2}^-} f(x) = 0$. One could also argue as follows: Since $f(x) = \cos x$ when $x \leq \frac{\pi}{2}$, $\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^-} \cos x = \lim_{x \to \frac{\pi}{2}} \cos x = \cos 0 = 0$, since $\cos x$ is continuous.

Therefore, $\lim_{x \to \frac{\pi}{2}^-} (x + f(x)) = \lim_{x \to \frac{\pi}{2}^-} x + \lim_{x \to \frac{\pi}{2}^-} f(x) = \dfrac{\pi}{2} + 0 = \dfrac{\pi}{2}$