Math 124 - Section 012
Test 1a

You have 50 minutes to complete the test. Read each problem carefully. Do not use decimal approximations unless you are explicitly asked to do so. ***Show all of your work.***

1. (10 pts) State the Intermediate Value Theorem. Draw a picture showing the important quantities in the theorem and the relationships between them.

2. (3 pts each) True or False: For each statement below, circle **TRUE** or **FALSE**. If the statement is false, give a counterexample; if the statement is true, briefly explain why it is true.

   a. **TRUE** or **FALSE**: Every polynomial is a rational function.

   b. **TRUE** or **FALSE**: $\log(AB) = \log(A) \cdot \log(B)$ for any positive numbers A and B.

   c. **TRUE** or **FALSE**: If $h(x)$ is an odd function and $h'(5) = 2$, then it must be the case that $h'(-5) = 2$.

   d. **TRUE** or **FALSE**: For any θ, $\arccos(\cos\theta) = \theta$.

3. (10 pts) A child rolls a rubber ball across a flat rooftop. After a few seconds, the ball drops off the edge of the roof and lands in a swimming pool, where it sinks to the bottom. If $h(t)$ represents the height of the ball above the ground t seconds after it leaves the child's hand, draw an approximate graph of $h'(t)$. Label the following points on the t-axis of your graph:

   a. A = The time at which the ball drops off the edge of the roof,

   b. B = The time at which the ball hits the surface of the swimming pool, and

   c. C = The time at which the ball hits the bottom of the swimming pool.

   (Hint: Draw the graph of $h(t)$ first.)

4. (10 pts) Suppose that, starting at the beginning of 2007, the price of gold were to increase by exactly 1.2% per day. How long would it take for the price of gold to triple? **Give an exact answer and a decimal approximation.**

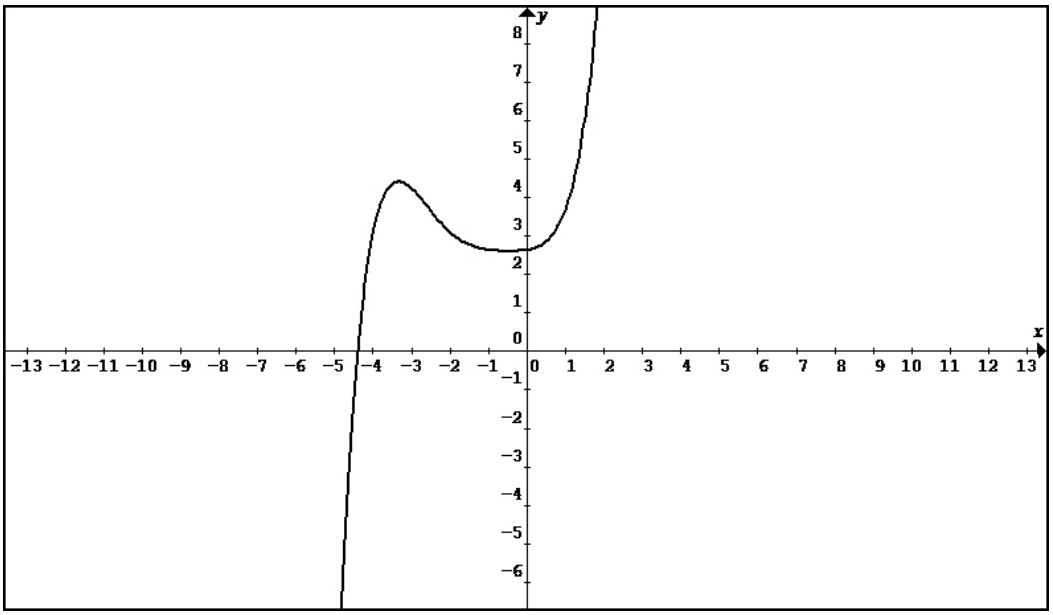
5. (10 pts) The temperature in a room is given by $C + A\sin(Bt)$ where t is measured in hours since midnight. Find values for A, B and C if the temperature in the room oscillates over a 24 hour period, the average temperature in the room is 73 degrees Fahrenheit, and the highest temperature the room reaches is 81 degrees Fahrenheit.

6. (10 pts) Let $g(x) = \frac{1}{x^2+2}$. Using the definition of the derivative, find $g'(0)$. (Write out all of the steps.)

7. Let

$$g(x) = \begin{cases} x+1 & \text{if } x \leq -1 \\ -(x+2)^2 + 1 & \text{if } x > -1 \end{cases},$$

let $h(x) = x^2 + 4x + 4$, and let $f(x)$ be the function graphed below.



For each of the following, compute the limit using the properties of limits described in class (write out all of the steps) or explain why it does not exist. You will need to approximate $f(-1)$ from the graph. (It may be helpful to draw the graph of $g(x)$.)

a. (5 pts) $\lim_{x \to -1^-} \dfrac{f(x)}{g(x)}$

b. (5 pts) $\lim_{x \to \infty} \frac{g(x)}{h(x)}$