

Mathematical Card Tricks

Teacher Guide

University of Arizona, SK Day 2014¹



A Magician Never Reveals Her Secrets...

... so be careful to not perform the same card trick multiple times in a row. Also, remember that audience misdirection and “patter” are just as important as the mathematical workings of the trick itself.

Another aspect is forcing a choice—the audience member thinks that the selection is her own, but really, the magician is directing all the steps.

The Math Behind the Magic

Keeping these principles in mind, it’s important to separate audience distraction from the trick itself.

- ♠ Beware of _____ *distractions* _____ .
- ◇ Watch for _____ *patterns* _____ .
- ♣ _____ *Break* _____ the problem down into _____ *smaller pieces* _____ .
- ♥ Work _____ *backwards* _____ .

¹Workshop by M. Hine, special thanks to M. Stone.
Image from http://www.freepik.com/free-vector/four-aces-poker-cards-clip-art_688687.htm

Other Questions:

How many possibilities of three sorts are there? Why?

Three choices of three options—first, middle, last. $3^3 = 27$.

How can we sort the piles differently if we want the chosen card be the first card in the final stack? The last card?

First, first, first. Last, last, last.

Why might sorting the chosen card to be the first card in the final stack be a bad idea when performing for an audience?

It's pretty easy for the audience to see this, especially if you repeat the trick!

Extensions:

What goes wrong if we sort middle-last-first? How could we fix the trick while keeping that same sort order?

We don't uniquely know the card (after the third sort it could still be any of three cards—map it out!)...but if you apply one more sort, then we know which card is the chose one.

How many sort options are there that let you determine the card position after exactly three sorts?

There are nine that work after three sorts. Map them out to find them all.

Create your own sorting algorithm to put the chosen card in a different spot.

The Final Three

Effect:

The audience chooses three cards. The magician deals out four piles of cards, but does not use all of the cards in the deck.

- ★ The first volunteer places one of the selected cards on top of the first pile, then places as many cards as she wishes from the second pile on top of her card.
- ★ The second volunteer places another of the selected cards on top of the (now smaller) second pile, then places as many cards as she wishes from the third pile on top of her card.
- ★ The third volunteer places the last of the selected cards on top of the (now smaller) third pile, then places as many cards as she wishes from the fourth pile on top of her card.

The magician gathers up all of the piles, and tells the audience to watch for their cards, interrupting the magician with “STOP!” when their card is shown. The magician alternately deals the cards into a face-up and a face-down pile. The magician repeats this process with the face-down pile until there are only three cards left. (There are three sorts—start with face-up every time.) The last three cards are the final cards!

Observations:

What patterns did you see? Was there anything special about how the magician directed the trick? What parts of the trick are important, and what parts are distractions?

The cutting of the cards by the audience doesn't actually change the number of cards between each of the chosen cards. The magician should also use a lot of patter. “Are you sure you didn't see your card?”

How it's done:

Let's work backwards...

SIMPLE VERSION— Deal 4 piles with 7 cards in each. It's pretty easy to see this working backward. What if we want to deal the last seven cards so that the chosen cards are the three downward-facing cards? How should the cards be arranged? (The last deal is

up – down – up – down – up – down – up

which means we need the pattern to be

Card – X – Card – X – Card – X – Card,

where the 'Card' denotes a filler card from the deck.) Now work backward twice more...

Extension:

Change the sorting algorithm and the pile sizes to make a new card trick.

Performance tip—have the spectator write the cards down, or choose cards that are easy to remember. For example, choose the Ace, King, Queen of Hearts.

FANCY Final Three²

Effect:

The audience chooses three cards. The magician deals out four piles of cards, using all of the remaining 52 cards in the deck.

- ★ The first volunteer places one of the selected cards on top of the first pile, then places as many cards as she wishes from the second pile on top of her card.
- ★ The second volunteer places another of the selected cards on top of the (now smaller) second pile, then places as many cards as she wishes from the third pile on top of her card.
- ★ The third volunteer places the last of the selected cards on top of the (now smaller) third pile, then places as many cards as she wishes from the fourth pile on top of her card.

The magician gathers up all of the piles, and tells the volunteers to watch for their cards, interrupting the magician with “STOP!” when their card is shown. The magician alternately deals the cards into a face-up and a face-down pile. The magician repeats this process with the face-down pile until there are only three cards left. (There are four sorts—start with face-up, start with face-up, start with face-down, start with face-up.) The last three cards are the final cards!

Observations:

What patterns did you see? Was there anything special about how the magician directed the trick? What parts of the trick are important, and what parts are distractions?

Again, the cutting of the cards by the audience doesn't actually change the number of cards between each of the chosen cards. The magician should also use a lot of patter. “Are you sure you didn't see your card?”

HINT—the order of the cards really matters, because there are not enough cards in the deck to make it symmetric!!! So working backwards is tricky to do this time, because each time you deal, the pile switches directions.

²<https://www.youtube.com/watch?v=fJ8VA9ICq8Q>

How it's done:

Let's work backwards. . .

FANCY VERSION— The dealer makes 4 piles, with 10-15-15-9 cards. Counting in groups of threes can make it look more random. It's easiest to count 10-15-9, then set the last 15 as a middle pile.

What number are the chosen cards after placing them in the piles? 10, 26, 42. Why? Why didn't cutting the deck matter? (Because cutting still leaves the order the same...think about how we collect the piles.) Then there are 4 rounds. First start with up, then start with up, third sort starts with down, and the fourth sort starts with up. The last three cards will be the final cards.

There's a really nice how-to video at <https://www.youtube.com/watch?v=fJ8VA9ICq8Q>

I solved this by drawing it out. Here's what I drew, with the cards kept after first sort marked with an X. We start with up.

	X		X		X		X		X		X	
X		X		X		X		X		X		X
	X		X		X		X		X		X	
X		X		X		X		X		X		X

The next sort starts with up again, but in reverse order. So start from the bottom left, and mark every other X card with an O.

	XO		X		XO		X		XO		X	
XO		X		XO		X		XO		X		XO
	X		XO		X		XO		X		XO	
X		XO		X		XO		X		XO		X

Reverse again, and start with a downward (i.e., kept) card, only sorting through the XO cards. Mark these with a P.

	XOP		X		XO		X		XOP		X	
XO		X		XOP		X		XO		X		XOP
	X		XO		X		XOP		X		XO	
X		XOP		X		XO		X		XOP		X

For the final sort, reverse again (i.e. start from the bottom left) and begin with an up card. Only sort through the XOP cards. Mark this with a Q.

	XOP		X		XO		X		XOPQ		X	
XO		X		XOP		X		XO		X		XOPQ
	X		XO		X		XOP		X		XO	
X		XOPQ		X		XO		X		XOP		X

The final three cards are located at spots 10, 26, and 42. This means the first card is the 10th card from the top, so the last pile the dealer makes should have 9 cards. Remember that we're stacking this backwards. There are 15 cards between each of the middle cards, and then 10 cards after the last card.

Other sorting patterns can be solved similarly. (E.g., sorting cards up-up-down, up-up-down with two sorts means the chosen cards should be located at spots 9, 27, and 45. So the piles should be 8-17-17-7.)

Performance tips—Have the spectator write the cards down, or choose cards that are easy to remember. Be very careful. One wrong move and you're finished! But a sense of humor rescues any situation. "And that's your card! Ta-da!" Move onto your next trick quickly. :)

Three Piles³

Effect:

The magician deals out piles of cards, using all or almost all 52 cards in the deck. She flips all of the piles over, then has a volunteer choose three piles. The magician gathers all of the other piles and any extra cards. The volunteer flips over the top card on two of the chosen piles. The magician quickly scans through the deck, and reveals the value of the top card on the third pile.

Observations:

What patterns did you see? Was there anything special about how the magician directed the trick? What parts of the trick are important, and what parts are distractions?

³A good explanation of the math behind this trick
<http://www.pleacher.com/mp/puzzles/tricks/piles3.html>

How it's done:

How can you encode the value of the card in the pile? Write an algebraic expression for each pile.

“Take a well shuffled deck of cards.

Deal out a card and add to its pip value enough cards to add to 13. (Jack = 11, Queen = 12, King = 13)

In other words, turn over the first card. Count out enough cards to 13. Then put the original card back on top of that pile face down.

Do the same for another pile and continue until you cannot deal another pile. Keep remaining cards in your hand.

Have someone pick any three of the piles. Return all other piles to the cards in your hand. Turn over the top cards on any 2 of the 3 piles. From the cards in the deck, deal out 10 cards plus a number equal to each of the turned up cards.

Count the number of cards left. Announce that number. Turn up the card on the top of the third pile.

Why does this work? Let's use some algebra to understand this trick.

Let the original face up card of Pile 1 be a .

Let the original face up card of Pile 2 be b .

Let the original face up card of Pile 3 be c .

In Pile 1, there are $13 - (a - 1)$ cards.

In Pile 2, there are $13 - (b - 1)$ cards.

In Pile 3, there are $13 - (c - 1)$ cards.

In your hand we have $52 - (14 - a + 14 - b + 14 - c)$ cards.

Simplifying, we get $52 - (42 - a - b - c) = 10 + a + b + c$.

So, when you count out $10 + a + b$ cards, you are left with c cards.”

(<http://www.pleacher.com/mp/puzzles/tricks/piles3.html>)

Extension:

Change the value of the piles to make a new trick.

Flourishes

♠ Mix it up!

- Make the trick look random. How?
- Cut/shuffle the deck. Better yet, have the audience help.
When can you do this for real without messing up the order?

How can you do this without messing up the order?

False cuts, false shuffles.

◇ Order matters

- Build audience expectation.
Is there a better order in which to present the tricks? Why?
*Presenting 21 before the Final Three conditions the audience that they *should* see their card.*

♣ Patter

- “Watch for your card.”
- “Am I going too fast?”
- “Are you sure?”

♥ Showmanship and the final reveal

- SIM-SALA-BIM
This is a way to count out 10 cards, one for each letter. The next card is the 11th card, which works well for 21.
- Florets
Make circles of 4 cards, remembering where you put the 11th card. Again this works for 21. Let the audience pick 2 piles, then 1 pile, then 2 cards, then one card, each time removing either the piles they chose, or those they did not choose, whichever leaves the chosen card on the table. It’s a forced choice, and works beautifully. Don’t repeat this, or else they’ll be on to you!

