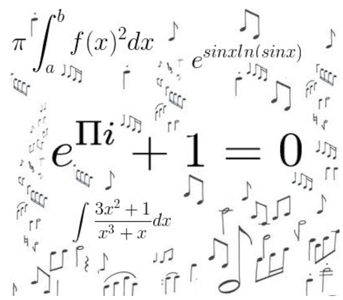
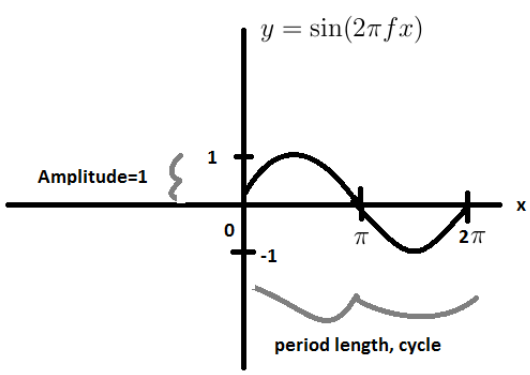
Math and Music



The Physics of Sound

Sound is generated when things vibrate. For example, speaking causes the air to vibrate, which passes the sound along to your ear drum. When your ear drum catches the vibration, you hear my voice.

Sound travels through the air in the form of waves. We can describe these waves using sine curves, $y = \sin(2\pi f x)$. The x can be any real number; usually we think of x as representing time. The period is the distance between x values before the cycle of the wave repeats again. It is often equal to 2π. The frequency f is the number of times the wave cycles within the normal period of 2π—how often the vibrations occur. The frequency is the reciprocal of the period.



mathwire.com
themusicmomma.blogspot.com

We can relate the speed at which sound travels to the wavelength and frequency of the wave using the following equation. We have that

$$W = \frac{c}{f}$$

where W is the wavelength, c is the speed of sound (approximately $345 m/s$), and f is the frequency of the wave.

We measure frequency in Hertz (Hz), which is equal to 1/second. This represents how many waves per second are generated by the sound. A typical human talks in a range of $100Hz$ to $3000Hz$. We use the term pitch to describe our perception of frequency. When we hear a high frequency, we say we hear a high pitch and similarly with a low frequency we hear a low pitch.

Calculation:

What is the range of wavelengths corresponding to the frequencies of human speech?

If you sing, many of you will be able to create sounds with the frequencies and wavelengths of a soprano. A soprano can create wavelengths between $132cm$ and $33cm$. What range of frequencies can she create?

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http://www.rhythmcreation.com/2007/11/15/frequency-and-wavelength-the-basics-of-what-sound-is-part-2

Wikepedia: Voice Range

Sound Intensity and Decibels

Sound intensity measures the strength of the experience of the sound. The distance you are from the sound has an inverse square relationship with the intensity.

For example, if the distance from the source is cut in half, then the intensity increases by a factor of 4. If you are 2 feet away from your friend who is shouting to you, then the intensity (how loud you think she sounds) will be quadrupled if you move 1 foot closer to your friend.

Humans have a high tolerance for intensity, so we use a logarithmic scale called decibels (dB) to measure it. Below are some noise levels you might recognize. Fill in the missing values.

Source	Intensity Level	How many times greater the intensity is than the threshold of hearing
Threshold of Hearing	$0dB$	10^0
Normal Conversation	$60dB$	10^6
Telephone dial tone	dB	10^8
Sun Chips Bag	$95dB$	$10^{9.5}$
Motorcycle	$100dB$	
Loud Rock Concert	$115dB$	
Violin	$82 - 92dB$	$10^{8.2} - 10^{9.2}$
Ear Pain Begins	dB	10^{13}
Military Jet Takeoff	$140dB$	10^{14}

Intensity has an interesting relationship with frequency. A $100Hz$ sound at $70dB$ will not sound as loud as a $1000Hz$ sound at $70dB$.

Question:

At the same intensity, which sounds louder: a low pitch or a high pitch?

http://www.physicsclassroom.com/class/sound/u11l2b.cfm
http://www.gcaudio.com/resources/howtos/loudness.html

Music

Instruments work exactly the same way as our voices. The instrument generates a wave, which travels through the air. Wind instruments like trumpets, trombones, and flutes work by blowing vibrating waves through pipes. The musician adjusts the length of the pipe and the speed of the air traveling through the pipe to change the pitch. String instruments like violins, cellos, and guitars work by stroking or strumming on tightened strings. The vibrations resonate (bounce around) inside the body of the instrument. On string instruments, we can actually see the vibration.

Violin Observations

Pay attention to how the sound changes with finger placement. Is there anything that you notice about the strings?

For stringed instruments, pitch depends on the thickness, tension, and length of the string.

Conclusions:

For each of the following, circle HIGHER or LOWER:

Tighter strings produce a HIGHER \ LOWER sound.

Thicker strings produce a HIGHER \ LOWER sound.

Longer strings produce a HIGHER \ LOWER sound.

Do these observations match your wavelength frequency calculations on the previous page?

http://method-behind-the-music.com/mechanics/strings

Math & Music

The middle A (the note we associate with tuning) has a frequency of $440Hz$.

Calculation:

Calculate the wavelength of middle A. If we shorten this wavelength, will that increase or decrease the frequency?

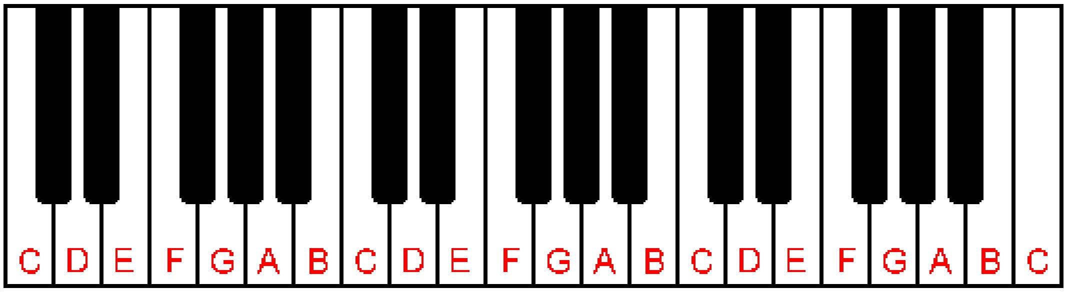
If we cut the frequency of middle A in half, we obtain the A to the left of the middle A (lower frequency).

Calculation:

Do you think this wavelength will be longer or shorter than our middle A? Calculate the new wavelength.

Music & Logarithms

We can visualize musical notes by looking at a piano. The distance between one A and the A immediately above it is called an octave.



As we calculated, the difference between two notes in an octave (think of the title notes of the song *Somewhere Over the Rainbow*) is that the higher note has twice the frequency of the lower note.

Brainstorm:

Look at the piano keyboard above. How many white keys are there between the one C to the next highest C, including the endpoints? (How many notes are in a scale?) Why would we call the difference between two consecutive C notes an octave?

How many white and black keys (also called semitones or half-steps) must you move up to get from one C to the next highest C?

How has the frequency changed from the first C to the second? What about from the first to the C above the second (the third C)?

Music & Logarithms

In Western music, we use an "equally tempered" scale. This means that we divide the difference in frequency logarithmically for each of the twelve semitones.

Calculation:

Come up with a formula describing the ratio of frequencies between two semitones.

Calculation:

Given that middle A has a frequency of $440Hz$, calculate the frequency of the E above it. (*Hint: count the number of semitones.*)

More Ratios

Other special ratios that we like to hear in music include fourths (up four tones in a scale– think of the theme from *Star Wars*) and fifths (think of *Here Comes the Bride*).

On the violin, the strings are tuned to major fifths.

On piano, we can see a major fifth by looking at the note C and the G directly above. A major fourth is the note C and the F directly above.

Calculation:

How many black and white keys do you touch to travel from a C to the G directly above it? This is a major fifth. What is the ratio of the frequencies in a major fifth?

How many black and white keys do you touch to travel from a C to the F directly above it? This is a major fourth. What is the ratio of the frequencies in a major fourth?

Let's experiment with changing wavelengths ourselves by constructing our own instruments.

Wind instruments like trumpets and trombones work by vibrating on the mouth piece while blowing air through the instrument. Wavelengths are still changed by increasing the pipe length.

We will build models of string instruments and models of wind instruments.

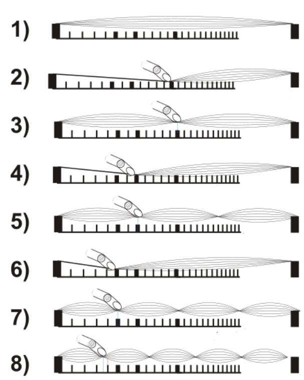
Building your string instrument:

1. Stretch a rubberband around your shoe box across the open portion of the box along the long side.

2. Pluck the rubberband and listen.

3. Hold the rubberband down in the center so it touches the bottom of the inside of the box and pluck one side of the rubberband and listen.

4. Repeat the previous step at one-fourth of the length of the rubberband.



Observations:

How does string length change the sound?

Building your wind instrument:

1. Select three straws.

2. Cut one straw in half, cut another straw into one-fourth and three-fourths.

3. Blow across the different straws with the same type of breath. How does the difference in sound relate to the length of the straw?

4. Use the masking tape to tape the different size straws next to each other in order from largest to smallest so that they are even at the top (they start at the same height).

5. Now you can play them together in different combinations to make music.

Observations:

How does straw length change the sound?