Section 7.1: Integration by Substitution

In this section, we will learn how to undo the chain rule. That is, we will learn how to find antiderivatives that are more or less of the form

$$\int g'(x) \cdot f(g(x))\, dx$$

To do this, we will generally be motivated to search for an "inside function", which we have written as $g(x)$ above, and to make the substitution $u = g(x)$. We will kick things off with an example.

Examples:

1. Find the indefinite integral $\int 3x^2 e^{x^3}\, dx$

The Method Of Substitution:

TO MAKE A SUBSTITUTION: Let u be the "inside function" and

$$du = u'(x)\,dx = \frac{du}{dx}\,dx$$

Examples:

2. Find the indefinite integrals.

   (a) $\displaystyle\int te^{t^2}\,dt$

(b) $\int 25e^{-0.2t}\,dt$

(c) $\int t\cos(t^2)\,dt$

(d) $\int x^2(1+2x^3)^2\,dx$

(e) $\int x(x^2-4)^{7/2}\,dx$

(f) $\displaystyle\int \frac{dy}{y+5}$

(g) $\displaystyle\int \sqrt{\cos(3t)}\,\sin(3t)\,dt$

(h) $\displaystyle\int \frac{x+1}{x^2+2x+19}\,dx$

Using Substitution To Compute Definite Integrals:

Either

- Compute the indefinite integral, expressing an antiderivative in terms of the original variable, and then evaluate the result at the original limits. Or
- Convert the original limits to new limits in terms of the original variable and do not convert the antiderivative back to the original variable.

Examples:

3. Compute the definite integrals.

   (a) $\displaystyle\int_0^{\pi/2} e^{-\cos\theta}\sin\theta\,d\theta$

   (b) $\displaystyle\int_1^4 \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$

4. Use the following table of values to evaluate the integral $\int_0^1 f'(x)\sin(f(x))\,dx$.

x	0	1	$\pi/2$	e	3
$f(x)$	5	7	8	10	11
$f'(x)$	2	4	6	9	12

5. Find the indefinite integral $\int \frac{1}{\sqrt{x+1}}\,dx$