Section 6.1: Continued

Finding absolute/global extrema on intervals that are not closed

We will now discuss how to go about finding absolute/global extrema of functions on intervals that may not be closed. We will first consider the case of a function defined on an open interval (a, b).

FINDING ABSOLUTE EXTREMA ON AN OPEN INTERVAL: To find the absolute extrema for a continuous function f defined on an open interval (a, b):

1. Find all critical numbers for f in (a, b).
2. Evaluate f for all critical numbers in (a, b).
3. Evaluate $\lim_{x \to a+} f(x)$ and $\lim_{x \to b-} f(x)$.
4. If either of the above (from step 3) is larger than the largest value found in step 2, there is no absolute maximum. Otherwise, the largest value found in step 2 is the absolute maximum of f on (a, b).
5. If either of the above (from step 3) is less than the least value found in step 2, there is no absolute minimum. Otherwise, the minimum value found in step 2 is the absolute minimum of f on (a, b).

Examples:

1. Find the absolute extrema of
$$f(x) = 2x + \frac{8}{x^2} + 1$$
on the interval $(0, \infty)$.

Suppose a function f is continuous on an open interval (a, b) and that f has exactly one critical number in the interval (a, b), located at $x = c$.

- If f has a relative maximum at $x = c$, then this relative maximum is the absolute maximum of f on the interval (a, b).
- If f has a relative minimum at $x = c$, then this relative minimum is the absolute minimum of f on the interval (a, b).

2. Find the absolute extrema of
$$f(x) = 12 - x + \frac{9}{x}$$
on the interval $(0, \infty)$, and indicate the x-values where they occur.

3. Find the absolute extrema of

$$f(x) = \frac{x}{x^2 + 1}$$

4. Suppose the cost function is given by

$$C(x) = 81x^2 + 17x + 324.$$

(a) Minimize the average cost function on the interval $[1, 10]$

(b) Minimize the average cost function on the interval $[10, 20]$