Section 5.2: Relative Extrema

RELATIVE EXTREMA: Let c be a number in the domain of the function f. Then $f(c)$ is a *relative (or local) maximum* for f if there exists an open interval (a, b) containing c such that

$$f(x) \leq f(c)$$

for all x in the interval (a, b). Likewise, $f(c)$ is a *relative (or local) minimum* for f if there exists an open interval (a, b) containing c such that

$$f(x) \geq f(c)$$

for all x in the interval (a, b). A function has a relative (or local) extremum (plural: extrema) at c if it has either a relative minimum or a relative maximum at c. If c is an endpoint of the domain of f, we only consider x in the half-open interval that is in the domain.

Basically, what were called "turning point" in an algebra class are now referred to as relative extrema.

Examples:

1. Find and classify the relative extrema for the function

$$f(x) = x^3 - 9x^2 - 48x + 52$$

2. Find and classify the relative extrema for the function

$$f(x) = x^4 - 4x^3$$

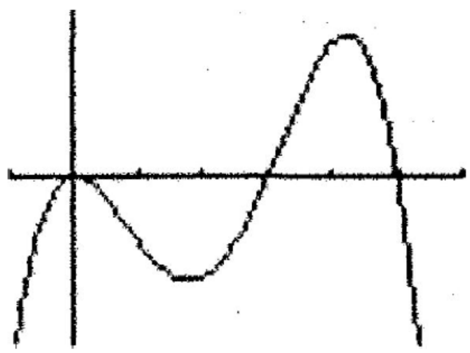
Let us now summarize what we have learned from the previous examples.

FIRST DERIVATIVE TEST: Let c be a critical number for a function f. Suppose that f is continuous on (a, b) and differentiable on (a, b) except possibly at c.

- $f(c)$ is a local maximum of f if $f'(x)$ changes from positive to negative at c.
- $f(c)$ is a local minimum of f if $f'(x)$ changes from negative to positive at c.

Warning: Not all critical numbers will be the locations of relative extrema. It is true that every relative extremum will be located at a critical number, but it is possible to find critical critical numbers for which we obtain neither a local maximum nor local minimum.

3. Below is a graph of f', the derivative of a function f. What are the critical points of the function f? Over what intervals is the function f increasing and decreasing? For what values of x does f have a local maximum or local minimum?



4. Find and classify the locations of all relative extrema for the following functions.

   (a) $f(x) = x^3 + 6x^2 + 9x - 8$

(b) $f(x) = \dfrac{x^2 - 6x + 8}{x + 2}$

(c) $f(x) = 3xe^x + 2$

5. Suppose that the cost function for a product is given by

$$C(x) = 0.002x^3 + 9x + 6912.$$

Find the production level (i.e. value of x) that will produce the minimum average cost per unit, $\overline{C}(x)$.

6. The demand equation for one type of computer networking system is

$$p = D(q) = 500qe^{-0.0016q^2},$$

where p is the price in dollars and q is the quantity of servers sold per month. Find the values of q and p that maximize revenue.