

BURSTING IN INTEGRATE-AND-FIRE NEURONAL NETWORKS

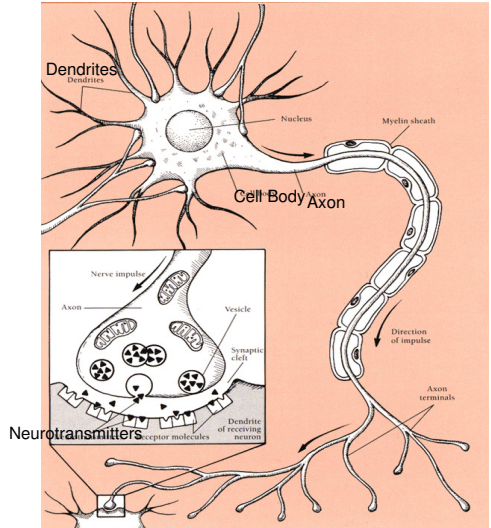
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Basic Biology of a Neuron

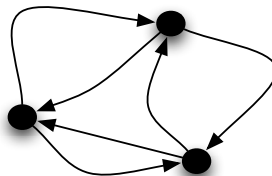
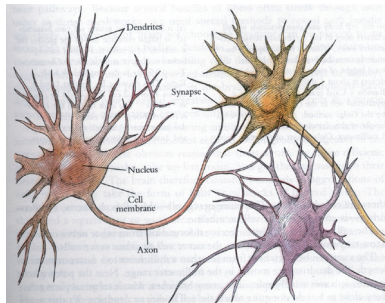
- ▶ **Dendrites** - receive input
- ▶ **Axon** - transmits output
- ▶ **Synapse** - junction between two neurons (pre-synaptic and post-synaptic neurons)
- ▶ **Neurotransmitters** - chemicals that carry signal across gap
- ▶ **Excitatory** - post-synaptic neuron more likely to fire
- ▶ **Inhibitory** - post-synaptic neuron less likely to fire



Neuronal Network Model

- ▶ Point-Neuron Model
 - ▶ Integrate-and-fire membrane potential dynamics
 - ▶ Poisson process external driving current

- ▶ Simple Network Model
 - ▶ Excitatory neurons
 - ▶ All-to-all connected
 - ▶ Instantaneous spike transmission

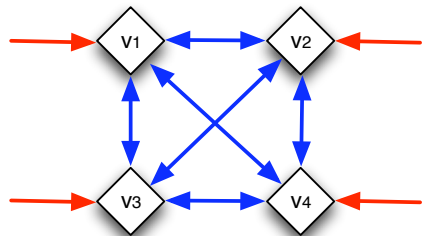


Excitatory Neuronal Network

All-to-all connected, current based, integrate-and-fire (IF) network

$$\frac{dv_i(t)}{dt} = -g_L(v_i(t) - V_R) + I_i(t), \quad i = 1, \dots, N$$

$$I_i(t) = f \sum_k \delta(t - t_{ik}) + \frac{S}{N} \sum_{j \neq i} \sum_k \delta(t - \tilde{t}_{jk})$$



$$V_R \leq v_i(t) < V_T$$

- ▶ V_R : reset potential (=0)
- ▶ V_T : firing threshold (=1)

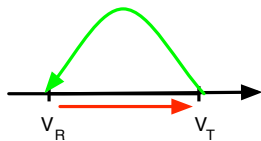
g_L : leakage rate (=1)

f and S : coupling strengths (> 0)

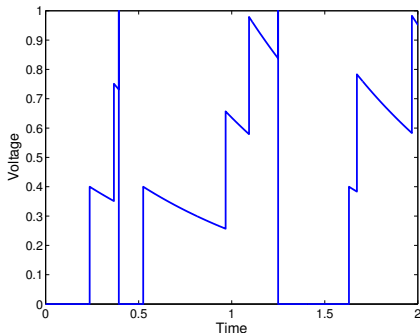
Integrate & Fire Dynamics

- ▶ $v_i(t)$ **evolves** according to the voltage ODE until \tilde{t}_{ik} , when $v_i(\tilde{t}_{ik}) \geq V_T$
- ▶ At \tilde{t}_{ik} , $v_i(t)$ is **reset** to V_R : $v_i(\tilde{t}_{ik}^+) = V_R$
- ▶ At \tilde{t}_{ik} , does not feel others spike
- ▶ S/N is **added** to the voltage, v_j , for $j = 1, \dots, N, j \neq i$

Reset:

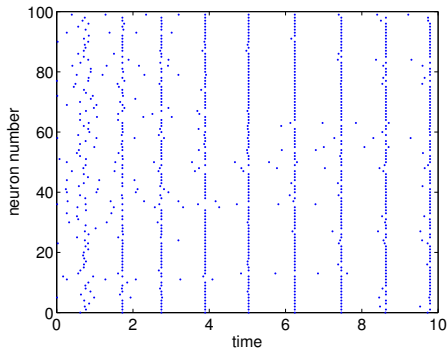
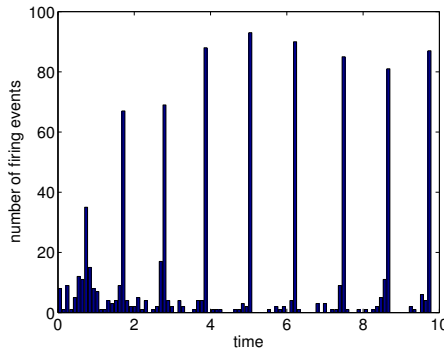


Membrane Potential:



Types of Synchronous Firing: Partial Synchrony

All-to-all connected network of $N = 100$ neurons

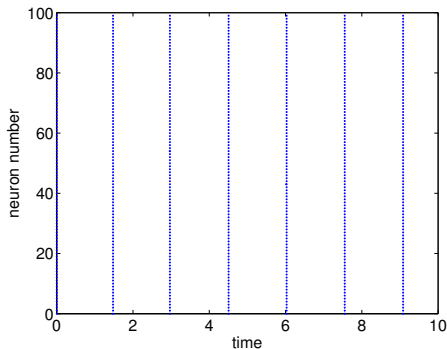
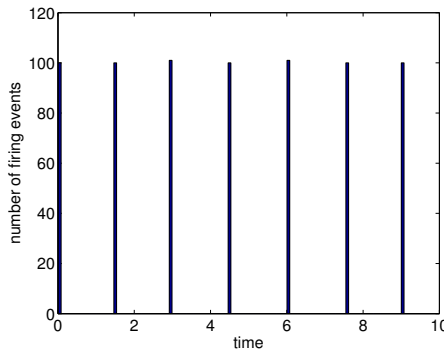


$$f = 0.01, f\nu = 1.2, S = 0.4$$

Types of Synchronous Firing: Perfect Synchrony

All-to-all connected network of $N = 100$ neurons

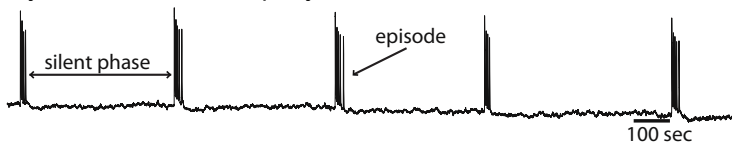
Consistent total firing events; "synchronizable network"



$$f = 0.001, f\nu = 1.2, S = 2.0$$

Bursting

Reality: a neuron fires rapidly



Simple Modifications:

- ▶ At \tilde{t}_{ik} , still feel other spikes
- ▶ Neuron has spiked n times at an instant
- ▶ $P = 1 \rightarrow P = e^{-n}$

Implications:

- ▶ $S > 1$ not runaway
- ▶ Neurons can spike $n > 1$ times at one instant
- ▶ Easy rapid firing of network

Definition

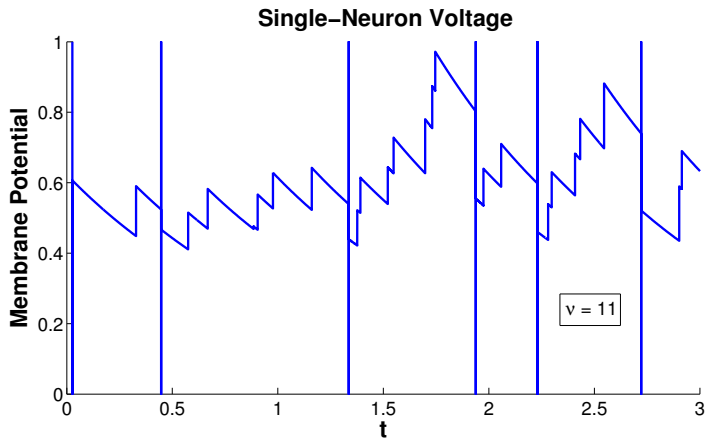
Burst: perfect synchronous firing event.

- ▶ What are the voltages distributions of neurons before and after a burst?
- ▶ How often do the bursts occur?
- ▶ What percentage of spikes are in bursts?
- ▶ How many times does each neuron fire within a burst?

Simulation Parameters:

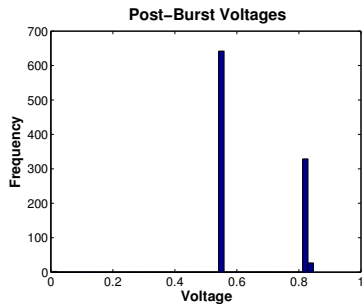
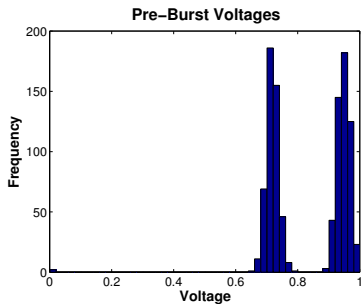
- ▶ Number of neurons: $N = 1000$
- ▶ External firing rate: $\nu = 1100$
- ▶ External firing strength: $f = 0.001$
- ▶ Coupling strength $S \in [1, \infty)$

Voltage of a Neuron



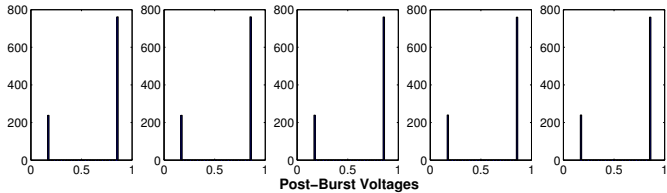
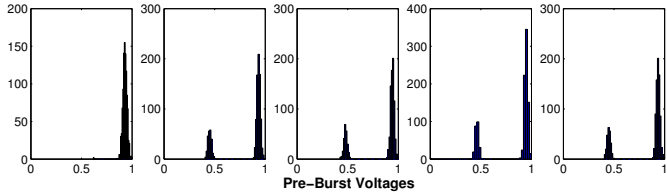
$$V_R \neq 0?$$

Voltages



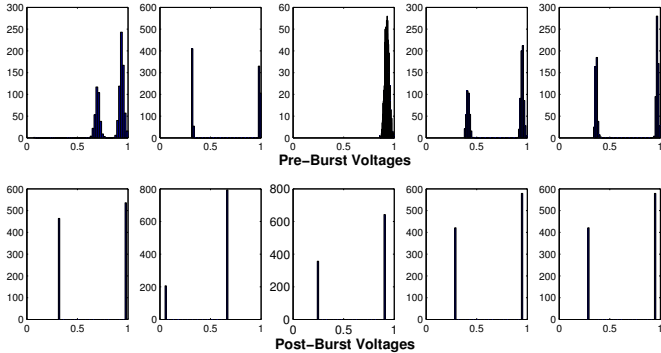
- ▶ Before: near $V_T = 1$
- ▶ After: depends on S

Voltages



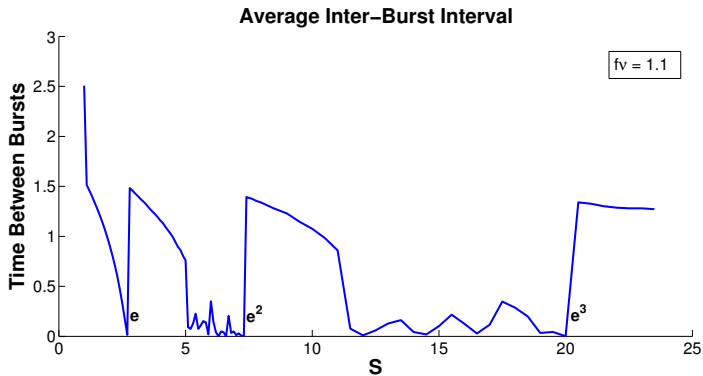
Steady state

Voltages



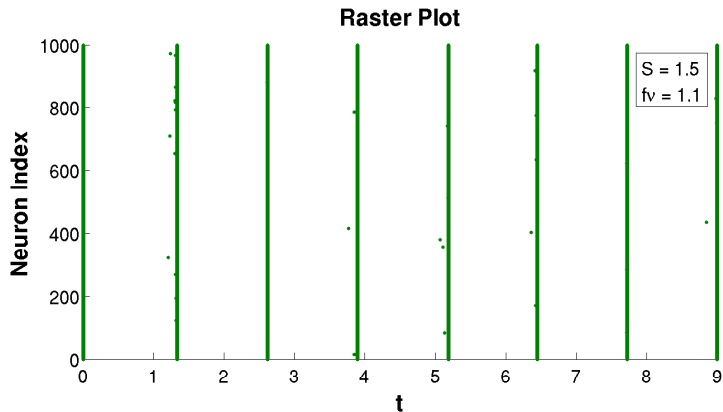
Steady state?

Average Inter-Burst Interval

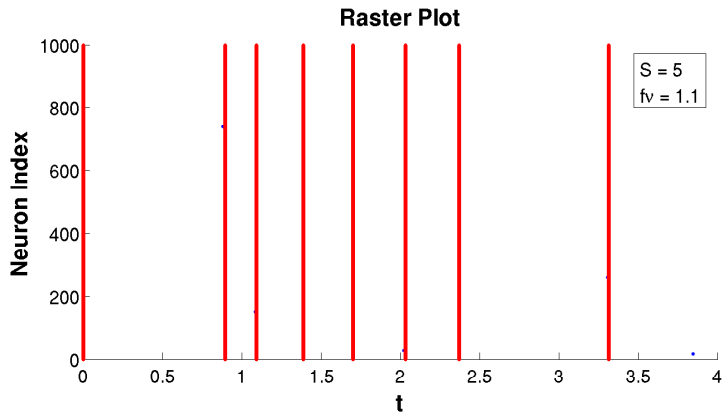


$$P = e^{-n}$$

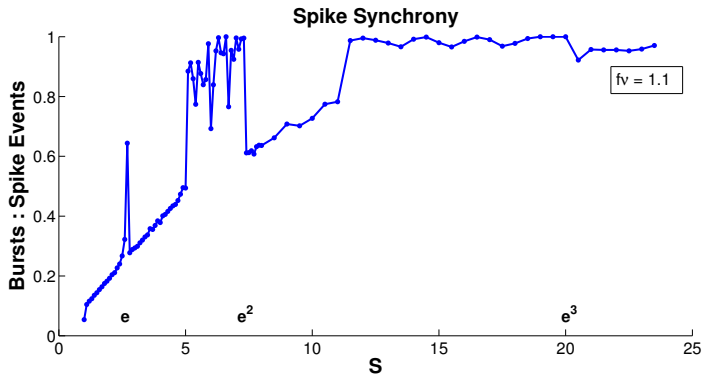
Raster Plots



Raster Plots



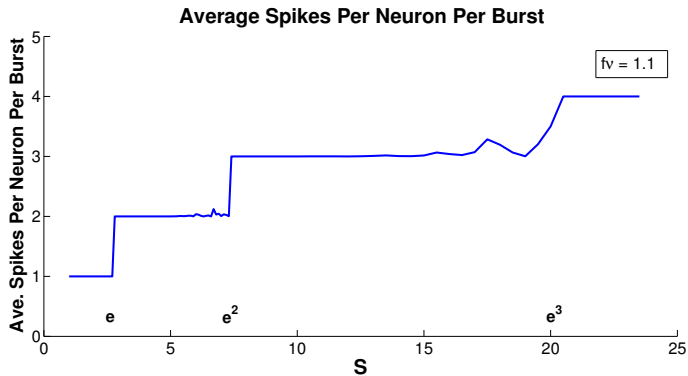
Spike Synchrony



$$P = e^{-n}$$

- ▶ Voltage ends high \rightarrow high synchrony
- ▶ S high \rightarrow high synchrony

Spikes Per Neuron Per Burst



$$P = e^{-n}$$

Conclusions

- ▶ Relaxed conditions to allow **bursting**
- ▶ Shut down bursting by $P = e^n$
- ▶ Looked at:
 - ▶ Neuron voltages
Voltage distributions may be reaching **steady state**
 - ▶ Frequency
Depends on post-burst voltages
 - ▶ Number of spikes per neuron
Step function
 - ▶ Spike synchrony
Generally increases with S
- ▶ Behavior shifts at e, e^2, e^3, \dots

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