

Quiz 6, Math 254, Section 001. Summer 2013.

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NAME: _____

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You will need to submit your calculations (on separate paper) along with your solution sheet. This work will be equivalent to two quizzes, and is due Tuesday at the beginning of class.

This extended exercise concerns the system of equations

$$3x' - 2y' + (3 - 2i)x + (3i - 2)y = 0, \quad (1)$$

$$4x' - y' + (4 - i)x + (4i - 1)y = 0. \quad (2)$$

A. Write the system using matrix notation, $\mathbf{A}\vec{x}' + \mathbf{B}\vec{x} = 0$:

$\mathbf{A} =$ _____, $\mathbf{B} =$ _____.

B. Find \mathbf{A}^{-1} and verify that your answer is correct by multiplying the two matrices together:

$\mathbf{A}^{-1} \cdot \mathbf{A} =$

C. Use your answer to part B. to express the system (1)-(2) in the form $\vec{x}' + \mathbf{C}\vec{x} = 0$.

D. By making the guess $\vec{x}(t) = \vec{v}e^{-rt}$, arrive at a linear algebra eigenvalue problem for the matrix \mathbf{C} , and find the eigenvalues and eigenvectors, and the general solution to the system of differential equations.

Bonus question: Apply the above procedure to the system of equations below. Show your work. You may use any resource to assist with finding eigenvalues as long as you (1) cite the relevant resource used and (2) verify by hand that you have found all the eigenvalues and eigenvectors.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}' + \begin{pmatrix} 2 & 2i & -1 \\ i & 1 & -i \\ 1 & i & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$