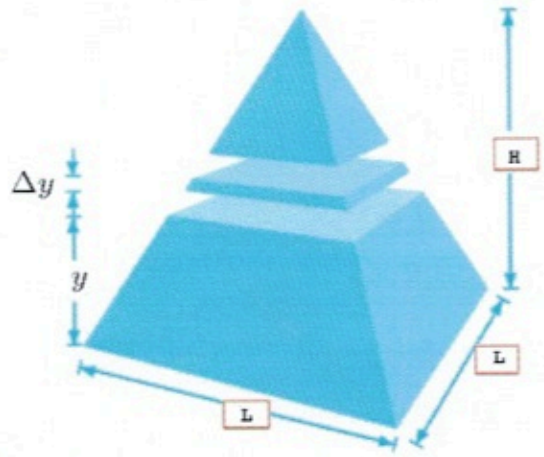
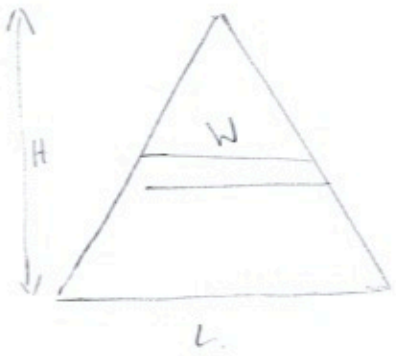
Math 129 – Exam 2 – Summer 2013 –

Print Name:

Signature:

(1) **25 points** Write a Riemann sum and then a definite integral representing the volume of the region, **using the slice shown**. Evaluate the integral exactly.





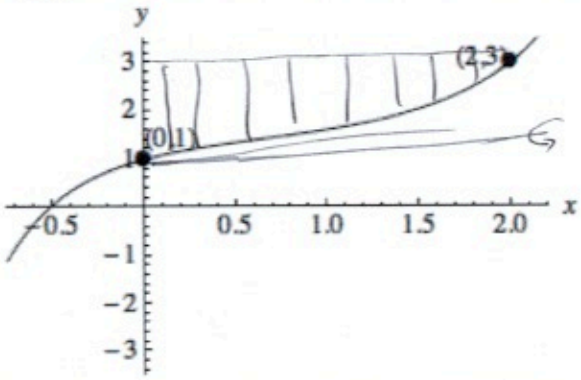
$$\frac{W}{H-y} = \frac{L}{H} \qquad W = \frac{L}{H}(H-y)$$

$$\text{Chunk volume} = \left(\frac{L}{H}(H-y)\right)^2 \Delta y$$

$$\text{Total volume} = \int_0^H \frac{L}{H}(H-y)^2\,dy$$

evaluate integral: $\frac{1}{3}L^2H$.

(2) **20 points** Consider the graph of $y = f(x)$ shown below. You do not need to find a formula for $f(x)$.



a) Shade the enclosed region bounded by the graphs of $y = f(x)$, and the lines $x = 0$, and $y = 3$.

b) Rotate the region in part a) around the line $y = 1$. **Set up** the integral(s) needed to find the volume of the resulting solid. Your answer will be in terms of $f(x)$.

$$\int_0^2 \pi\left[(3-1)^2 - (f(x)-1)^2\right] dx.$$

(3) **20 points** My coffee mug is cylindrical with a 3cm radius, and a 10cm height. When it is full, the density ρ of sugar (in gm/cm^3) in my coffee, as a function of the height, h, above the bottom of the mug, is given by the formula $\rho(h) = 0.01(10 - h)$. Write down a Riemann sum, and then derive an integral that gives the exact amount of sugar in my coffee mug.

chunk vol = $\pi(9)\,\Delta h$.

sugar content: $(0.01)(10-h)\,9\pi\,\Delta h$

Total $9\pi \int_0^{10} (0.01)(10-h)\,dh$

(4) **15 points** True or False Questions. No explanations needed, and no partial credit.

A. A city occupies a region in the xy-plane, with population density $\delta(y) = 1 + y$. To set up an integral representing the total population in the city, we should slice the region into strips parallel to the ~~x-axis~~.

TRUE **FALSE**

B. A speeding train is heading directly at a car which has stalled on the train tracks. Using superhuman strength, a superhero stops the train before the collision happens. By the scientific definition, the work superman did was positive.

TRUE **FALSE**

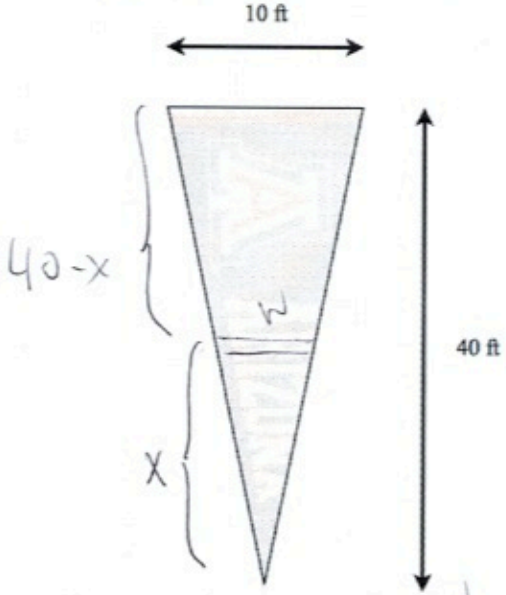
C. The population density in a circular city of radius 2 depends on the distance r from the center by $f(r) = 10 - 3r$, so that the density is greatest at the center. **True or False**: The population of the inner city, $0 \leq r \leq 1$, is greater than the population of the suburbs, $1 \leq r \leq 2$.

TRUE **FALSE**

$$\int_0^1 2\pi r(10-3r)\,dr = 8\pi$$

$$\int_1^2 2\pi r(10-3r)\,dr = 16\pi.$$

(5) **20 points** A giant triangular Arizona Wildcat flag is hanging off the roof of the football stadium, and students from ASU are planning to steal it. The flag has mass density $f(x)$ lbs per square foot, where x is measured from the bottom point of the flag. The width across the top is 10 feet, and the flag is 40 feet long. **Set up** an integral representing the work required to pull the flag onto the roof of the stadium.



$$\frac{w}{x} = \frac{10}{40}$$

$$w = \frac{1}{4}x$$

chunk area $= \frac{1}{4}x \cdot \Delta x$

weight of chunk $f(x)\frac{1}{4}x\,\Delta x$ lbs

work $(40-x)f(x)\left(\frac{1}{4}x\right)\Delta x$

$$\text{Total Work} = \int_0^{40} (40-x)f(x)\left(\frac{1}{4}x\right)dx.$$