

NAME : \_\_\_\_\_

## Exam 2, Math 254, Summer 2013

23 July 2013

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### INSTRUCTIONS:

- This is a closed book, closed notes exam.
- You are not to give or receive help from any outside source during the exam.
- You have **1 hour**.

2. (20) Find a solution of the differential equation.

$$\frac{d^2y}{dx^2} - 9y = 5x^2$$

$$y = -\frac{5}{9}x^2 - \frac{10}{81}$$

3. (20) What is the correct guess to apply the method of undetermined coefficients and find a solution to each of the following differential equations? **NOTE: You do not need to solve these differential equations.**

A.  $y'' - 6y' + 9y = te^{3t}$ ,

aux. eqn:  $r^2 - 6r + 9 = 0$   
 $(r-3)^2 = 0$   
 double root.

So the guess is  
 $y = t^2(A+B)e^{3t}$ .

B.  $y'' - 4y = t + \sin(2t)$

aux. eqn.  $r^2 - 4 = 0$   
 $r = \pm 2$

So the guess is  $y = At + B + C \cos 2t + D \sin 2t$

4. (20) Solve the initial value problem

$$y'' + 4y = \cos t, \quad y(0) = 0, \quad y'(0) = 3.$$

homogeneous:

$$Y_h = C_1 \cos 2t + C_2 \sin 2t$$

particular:

$$Y_p = \frac{1}{3} \cos t$$

full soln.  
w/ Initial  
values:

$$Y = \frac{1}{3} \cos t + \frac{3}{2} \sin 2t - \frac{1}{3} \cos(2t)$$

5. (20) Consider the *third order, constant coefficient* differential equation  $y''' - 2y'' + y' = 0$ .

A. Find and solve the auxiliary equation for this differential equation.

$$y = e^{rt} \quad r^3 - 2r^2 + r = 0 \quad r(r-1)(r-1) = 0$$
$$r = 0, \quad r = 1, \quad r = 1 \quad (\text{double root})$$

B. What is the general solution to this differential equation? (Your answer should contain three arbitrary constants.)

$$y = c_1 + c_2 e^t + c_3 t e^t.$$