Polynomial Division using the Box Method, when you have a nonzero remainder

Matt Bush, Math 120R, Fall 2011

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- Use the arrow keys or mouse wheel to move from slide to slide.

Before you go any further in this document, FIRST watch this youtube video, which is where I got the idea to create this (thanks to Camille for showing me the video)

▸ http://www.youtube.com/watch?v=V-Q6jBYn3Oc

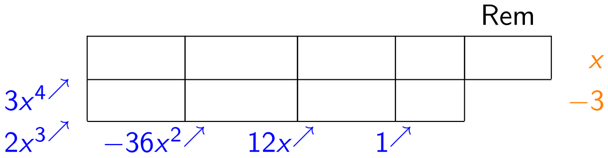
If the link doesn't work for you, try hitting ESC to leave full-screen mode, or just copy-pasting the link manually. Here it is again: http://www.youtube.com/watch?v=V-Q6jBYn3Oc

- The method described in the video works great when the remainder of your division is zero, but it requires a small modification when the remainder is nonzero. Here I present two worked-out examples where the remainder is nonzero.

- The method described in the video works great when the remainder of your division is zero, but it requires a small modification when the remainder is nonzero. Here I present two worked-out examples where the remainder is nonzero.

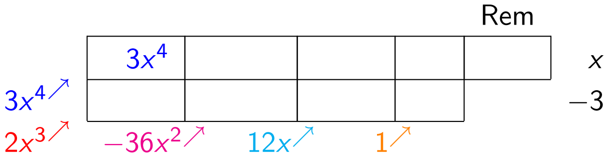
- You are welcome to use this method, polynomial long division, or synthetic division (as long as you're dividing by a linear factor) on the test. If you have some other method that you like to use, please tell me about it beforehand.

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:



- Write $3x^4 + 2x^3 - 36x^2 + 12x + 1$ along the diagonals of the box
- Write $x - 3$ on the right hand side.

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:



- I'm changing the colors of $3x^4 + 2x^3 - 36x^2 + 12x + 1$ so that each term is a different color, so that you can see what lines up with what.
- This upper-left box is the only box that can have an x^4 term in it, so it has to equal the leading term of the numerator, which is $3x^4$.

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:

	$3x^3$				Rem	
	$3x^4$					x
$3x^4 \nearrow$						-3
$2x^3 \nearrow$	$-36x^2 \nearrow$	$12x \nearrow$	$1 \nearrow$			

We divide the $3x^4$ by x from the far right, to get $\dfrac{3x^4}{x} = 3x^3$.

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:

	$3x^3$				Rem	
	$3x^4$					x
$3x^4 \nearrow$	$-9x^3$					-3
$2x^3 \nearrow$	$-36x^2 \nearrow$	$12x \nearrow$	$1 \nearrow$			

Now we multiply $3x^3$ (from on top) by -3 (from the far right).

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:

	$3x^3$				Rem	
	$3x^4$	$11x^3$				x
$3x^4 \nearrow$	$-9x^3$					-3
$2x^3 \nearrow$	$-36x^2 \nearrow$	$12x \nearrow$	$1 \nearrow$			

We already have -9 x^3's, and we need 2 of them, so we have to add 11 more.

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:

	$3x^3$	$11x^2$			Rem	
	$3x^4$	$11x^3$				x
$3x^4 \nearrow$	$-9x^3$					-3
$2x^3 \nearrow$	$-36x^2 \nearrow$	$12x \nearrow$	$1 \nearrow$			

We divide the $11x^3$ by x from the far right, to get $\frac{11x^3}{x} = 11x^2$.

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:

	$3x^3$	$11x^2$			Rem	
	$3x^4$	$11x^3$				x
$3x^4$ ↗	$-9x^3$	$-33x^2$				-3
$2x^3$ ↗	$-36x^2$ ↗	$12x$ ↗	1 ↗			

Now we multiply $11x^2$ (from on top) by -3 (from the far right).

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:

	$3x^3$	$11x^2$			Rem	
	$3x^4$	$11x^3$	$-3x^2$			x
$3x^4 \nearrow$	$-9x^3$	$-33x^2$				-3
$2x^3 \nearrow$	$-36x^2 \nearrow$	$12x \nearrow$	$1 \nearrow$			

We already have -33 x^2's, and we need -36 of them, so we have to add -3 (or take away 3) more.

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:

	$3x^3$	$11x^2$	$-3x$		Rem	
	$3x^4$	$11x^3$	$-3x^2$			x
$3x^4 \nearrow$	$-9x^3$	$-33x^2$				-3
$2x^3 \nearrow$	$-36x^2 \nearrow$	$12x \nearrow$	$1 \nearrow$			

We divide the $-3x^2$ by x from the far right, to get $\frac{-3x^2}{x} = -3x$.

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:

	$3x^3$	$11x^2$	$-3x$		Rem	
	$3x^4$	$11x^3$	$-3x^2$			x
$3x^4 \nearrow$	$-9x^3$	$-33x^2$	$9x$			-3
$2x^3 \nearrow$	$-36x^2 \nearrow$	$12x \nearrow$	$1 \nearrow$			

Now we multiply $-3x$ (from on top) by -3 (from the far right).

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:

	$3x^3$	$11x^2$	$-3x$		Rem	
	$3x^4$	$11x^3$	$-3x^2$	$3x$		x
$3x^4\nearrow$	$-9x^3$	$-33x^2$	$9x$			-3
$2x^3\nearrow$	$-36x^2\nearrow$	$12x\nearrow$	$1\nearrow$			

We already have 9 x's, and we need 12 of them, so we have to add 3 more.

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:

	$3x^3$	$11x^2$	$-3x$	3	Rem	
	$3x^4$	$11x^3$	$-3x^2$	$3x$		x
$3x^4 \nearrow$	$-9x^3$	$-33x^2$	$9x$			-3
$2x^3 \nearrow$	$-36x^2 \nearrow$	$12x \nearrow$	$1 \nearrow$			

We divide the $3x$ by x from the far right, to get $\dfrac{3x}{x} = 3$.

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:

	$3x^3$	$11x^2$	$-3x$	3	Rem	
	$3x^4$	$11x^3$	$-3x^2$	$3x$		x
$3x^4 \nearrow$	$-9x^3$	$-33x^2$	$9x$	-9		-3
$2x^3 \nearrow$	$-36x^2 \nearrow$	$12x \nearrow$	$1 \nearrow$			

Now we multiply 3 (from on top) by -3 (from the far right).

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:

	$3x^3$	$11x^2$	$-3x$	3	Rem	
	$3x^4$	$11x^3$	$-3x^2$	$3x$	10	x
$3x^4 \nearrow$	$-9x^3$	$-33x^2$	$9x$	-9		-3
$2x^3 \nearrow$	$-36x^2 \nearrow$	$12x \nearrow$	$1 \nearrow$			

We already have -9 1's, and we need 1 of them, so we have to add 10 more.

Dividing $2x^4 + 5x^3 - 22x^2 - 3x + 2$ by $x - 3$ using the box method:

	$3x^3$	$11x^2$	$-3x$	3	Rem	
	$3x^4$	$11x^3$	$-3x^2$	$3x$	10	x
$3x^4 \nearrow$	$-9x^3$	$-33x^2$	$9x$	-9		-3
$2x^3 \nearrow$	$-36x^2 \nearrow$	$12x \nearrow$	$1 \nearrow$			

Now we have our answer:

$$3x^4 + 2x^3 - 36x^2 + 12x + 1 = (3x^3 + 11x^2 - 3x + 3)(x - 3) + (10)$$

Dividing $2x^4+5x^3-22x^2-3x+2$ by $x-3$ using the box method:

	$3x^3$	$11x^2$	$-3x$	3	Rem	
	$3x^4$	$11x^3$	$-3x^2$	$3x$	10	x
$3x^4\nearrow$	$-9x^3$	$-33x^2$	$9x$	-9		-3
$2x^3\nearrow$	$-36x^2\nearrow$	$12x\nearrow$	$1\nearrow$			

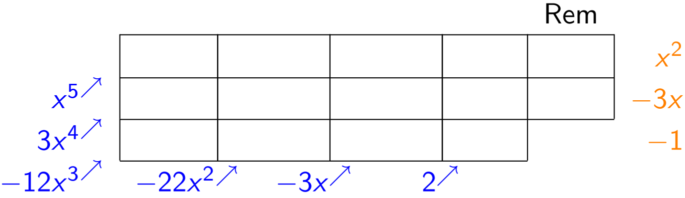
Now we have our answer:

$$3x^4+2x^3-36x^2+12x+1=(3x^3+11x^2-3x+3)(x-3)+(10)$$

or equivalantly,

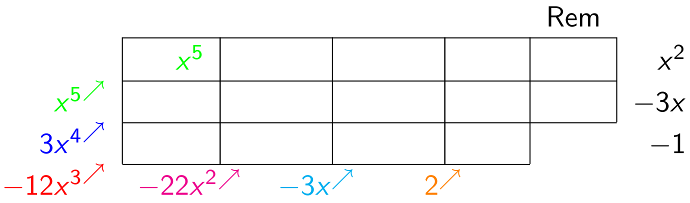
$$\frac{3x^4+2x^3-36x^2+12x+1}{x-3}=(3x^3+11x^2-3x+3)+\frac{10}{x-3}$$

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:



- Write $x^5 + 3x^4 - 12x^3 - 10x^2 - 3x + 2$ along the diagonals of the box
- Write $x^2 - 3x - 1$ on the right hand side.

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:



- I'm changing the colors of $x^5 + 3x^4 - 12x^3 - 10x^2 - 3x + 2$ so that each term is a different color, so that you can see what lines up with what.
- This upper-left box is the only box that can have an x^5 term in it, so it has to equal the leading term of the numerator, which is x^5.

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3				Rem	
x^5 ↗	x^5					x^2
$3x^4$ ↗						$-3x$
$-12x^3$ ↗						-1
	$-22x^2$ ↗	$-3x$ ↗	2 ↗			

We divide the green box by x^2 from the far right, to get $\frac{x^5}{x^2} = x^3$.

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3				Rem	
	x^5					x^2
$x^5 \nearrow$	$-3x^4$					$-3x$
$3x^4 \nearrow$	$-x^3$					-1
$-12x^3 \nearrow$	$-22x^2 \nearrow$	$-3x \nearrow$	$2 \nearrow$			

Now we multiply x^3 (from on top) by $-3x$ and -1 (from the far right).

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3				Rem	
	x^5	$6x^4$				x^2
$x^5\nearrow$	$-3x^4$					$-3x$
$3x^4\nearrow$	$-x^3$					-1
$-12x^3\nearrow$	$-22x^2\nearrow$	$-3x\nearrow$	$2\nearrow$			

We already have -3 x^4's, and we need 3 of them, so we have to add 6 more.

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3	$6x^2$			Rem	
	x^5	$6x^4$				x^2
$x^5 \nearrow$	$-3x^4$					$-3x$
$3x^4 \nearrow$	$-x^3$					-1
$-12x^3 \nearrow$	$-22x^2 \nearrow$	$-3x \nearrow$	$2 \nearrow$			

We divide the blue $6x^4$ by x^2 from the far right, to get $\frac{6x^4}{x^2} = 6x^2$.

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3	$6x^2$			Rem	
	x^5	$6x^4$				x^2
$x^5\nearrow$	$-3x^4$	$-18x^3$				$-3x$
$3x^4\nearrow$	$-x^3$	$-6x^2$				-1
$-12x^3\nearrow$	$-22x^2\nearrow$	$-3x\nearrow$	$2\nearrow$			

Now we multiply $6x^2$ (from on top) by $-3x$ and -1 (from the far right).

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3	$6x^2$			Rem	
	x^5	$6x^4$	$6x^3$			x^2
$x^5 \nearrow$	$-3x^4$	$-18x^3$				$-3x$
$3x^4 \nearrow$	$-x^3$	$-6x^2$				-1
$-12x^3 \nearrow$	$-22x^2 \nearrow$	$-3x \nearrow$	$2 \nearrow$			

We already have $-18 - 1 = -19$ x^3's, and we need -12 of them, so we have to add 6 more.

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3	$6x^2$	$6x$		Rem	
	x^5	$6x^4$	$6x^3$			x^2
$x^5 \nearrow$	$-3x^4$	$-18x^3$				$-3x$
$3x^4 \nearrow$	$-x^3$	$-6x^2$				-1
$-12x^3 \nearrow$	$-22x^2 \nearrow$	$-3x \nearrow$	$2 \nearrow$			

We divide the red $6x^3$ by x^2 from the far right, to get $\frac{6x^3}{x^2} = 6x$.

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3	$6x^2$	$6x$		Rem	
	x^5	$6x^4$	$6x^3$			x^2
$x^5 \nearrow$	$-3x^4$	$-18x^3$	$-18x^2$			$-3x$
$3x^4 \nearrow$	$-x^3$	$-6x^2$	$-6x$			-1
$-12x^3 \nearrow$	$-22x^2 \nearrow$	$-3x \nearrow$	$2 \nearrow$			

Now we multiply $6x$ (from on top) by $-3x$ and -1 (from the far right).

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3	$6x^2$	$6x$		Rem	
	x^5	$6x^4$	$6x^3$	$2x^2$		x^2
$x^5 \nearrow$	$-3x^4$	$-18x^3$	$-18x^2$			$-3x$
$3x^4 \nearrow$	$-x^3$	$-6x^2$	$-6x$			-1
$-12x^3 \nearrow$	$-22x^2 \nearrow$	$-3x \nearrow$	$2 \nearrow$			

We already have $-18 - 6 = -24$ x^2's, and we need -22 of them, so we have to add 2 more.

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3	$6x^2$	$6x$	2	Rem	
	x^5	$6x^4$	$6x^3$	$2x^2$		x^2
$x^5 \nearrow$	$-3x^4$	$-18x^3$	$-18x^2$			$-3x$
$3x^4 \nearrow$	$-x^3$	$-6x^2$	$-6x$			-1
$-12x^3 \nearrow$	$-22x^2 \nearrow$	$-3x \nearrow$	$2 \nearrow$			

We divide the magenta $2x^2$ by x^2 from the far right, to get $\dfrac{2x^2}{x^2} = 2$.

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3	$6x^2$	$6x$	2	Rem	
	x^5	$6x^4$	$6x^3$	$2x^2$		x^2
$x^5\nearrow$	$-3x^4$	$-18x^3$	$-18x^2$	$-6x$		$-3x$
$3x^4\nearrow$	$-x^3$	$-6x^2$	$-6x$	-2		-1
$-12x^3\nearrow$	$-22x^2\nearrow$	$-3x\nearrow$	$2\nearrow$			

Now we multiply 2 (from on top) by $-3x$ and -1 (from the far right).

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3	$6x^2$	$6x$	2	Rem	
	x^5	$6x^4$	$6x^3$	$2x^2$	$9x$	x^2
$x^5\nearrow$	$-3x^4$	$-18x^3$	$-18x^2$	$-6x$		$-3x$
$3x^4\nearrow$	$-x^3$	$-6x^2$	$-6x$	-2		-1
$-12x^3\nearrow$	$-22x^2\nearrow$	$-3x\nearrow$	$2\nearrow$			

We already have $-6 - 6 = -23$ x's, and we need -3 of them, so we have to add 9 more.

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3	$6x^2$	$6x$	2	Rem	
	x^5	$6x^4$	$6x^3$	$2x^2$	$9x$	x^2
$x^5 \nearrow$	$-3x^4$	$-18x^3$	$-18x^2$	$-6x$	4	$-3x$
$3x^4 \nearrow$	$-x^3$	$-6x^2$	$-6x$	-2		-1
$-12x^3 \nearrow$	$-22x^2 \nearrow$	$-3x \nearrow$	$2 \nearrow$			

We already have -2 1's, and we need 2 of them, so we have to add 4 more.

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3	$6x^2$	$6x$	2	Rem	
	x^5	$6x^4$	$6x^3$	$2x^2$	$9x$	x^2
$x^5 \nearrow$	$-3x^4$	$-18x^3$	$-18x^2$	$-6x$	4	$-3x$
$3x^4 \nearrow$	$-x^3$	$-6x^2$	$-6x$	-2		-1
$-12x^3 \nearrow$	$-22x^2 \nearrow$	$-3x \nearrow$	$2 \nearrow$			

Now we have our answer:

$$x^5+3x^4-12x^3-10x^2-3x+2=(x^3+6x^2+6x+2)(x^2-3x-1)+(9x+4)$$

Dividing $x^5 + 3x^4 - 12x^3 - 22x^2 - 3x + 2$ by $x^2 - 3x - 1$ using the box method:

	x^3	$6x^2$	$6x$	2	Rem	
	x^5	$6x^4$	$6x^3$	$2x^2$	$9x$	x^2
$x^5\nearrow$	$-3x^4$	$-18x^3$	$-18x^2$	$-6x$	4	$-3x$
$3x^4\nearrow$	$-x^3$	$-6x^2$	$-6x$	-2		-1
$-12x^3\nearrow$	$-22x^2\nearrow$	$-3x\nearrow$	$2\nearrow$			

Now we have our answer:

$$x^5+3x^4-12x^3-10x^2-3x+2=(x^3+6x^2+6x+2)(x^2-3x-1)+(9x+4)$$

or equivalantly,

$$\frac{x^5+3x^4-12x^3-10x^2-3x+2}{x^2-3x-1}=(x^3+6x^2+6x+2)+\frac{9x+4}{x^2-3x-1}$$