Math120R: Precalculus Final Exam Review
Summer II 2011

Note: This study aid is intended to help you review for the final exam. It covers the primary concepts in the course. Do not expect this review to be identical to the actual final exam. You should also review tests, notes, and homework problems given throughout the semester.

Multiple Choice Practice

1. Which of the following equations determine y as a function of x?

(1) $3x + 2y^3 = 10$ (2) $\sqrt{x+1} + y = 8$ (3) $2x - y^2 - 7 = 0$

(4) $3x^2 - xy = 1$

(A) all of them (B) 1 and 3 only (C) 1, 2, and 4 only

(D) 1 and 2 only (E) 1 and 4 only

2. If $f(t) = 3t^2 - 2$, find $f(k-2)$.

(A) $3k^2 - 12k + 12$ (B) $3k^2 - 8$ (C) $3k^2 - 4k + 2$

(D) $3k^2 - 4$ (E) None of these

3. If $f(x) = \begin{cases} -x+4 & \text{if } x < 1 \\ x^2 & \text{if } 1 < x \le 3 \\ \sqrt{x+3} & \text{if } x > 3 \end{cases}$, what is $f(-2)$?

(A) 6 (B) 4 (C) 2 (D) 1 (E) None of these

4. Express the area of a rectangle as a function of its width if the width is 25% of its length. Let L and W represent length and width respectively.

(A) $A = (.25W)(W)$ (B) $A = (.75W)(W)$ (C) $A = (4W)(W)$

(D) $A = 4LW$ (E) None of these

5. The equation of the line passing through the ordered pair $(a,0)$ and parallel to the line $x + 2y = 7$ is:

(A) $y = -\frac{1}{2}x + a$ (B) $y = -\frac{1}{2}x + \frac{a}{2}$ (C) $y = \frac{1}{2}x + a$

(D) $y = \frac{1}{2}x + \frac{a}{2}$ (E) None of these

6. An equation of a graph obtained from vertically compressing the graph of $y = \sqrt{x}$ then shifting the graph up twenty units is:

(A) $y = \frac{5}{3}\sqrt{x} + 20$ (B) $y = \sqrt{\frac{7}{2}x + 20}$ (C) $y = \frac{3}{4}\sqrt{x} + 20$

(D) $y = 2\sqrt{x + 20}$ (E) None of these

7. You can get the graph of $y = -g(2x)$ by transforming the graph of $y = g(x)$ in the following way:

(A) Compress horizontally and reflect across the x-axis.

(B) Compress horizontally and reflect across the y-axis.

(C) Expand vertically and reflect across the x-axis.

(D) Expand vertically and reflect across the y-axis.

(E) None of these

8. If $(-4,7)$ is a point on the graph of $y = h(t)$, which of the following must be a point on the graph of $y = h(-t) - 2$?

(A) $(-4,-9)$ (B) $(-4,-5)$ (C) $(4,5)$ (D) $(4,9)$ (E) None of these

9. Find the vertex of the quadratic function $f(x) = \frac{4}{7}x^2 - \frac{16}{7}x + 3$. The y-coordinate of the vertex is:

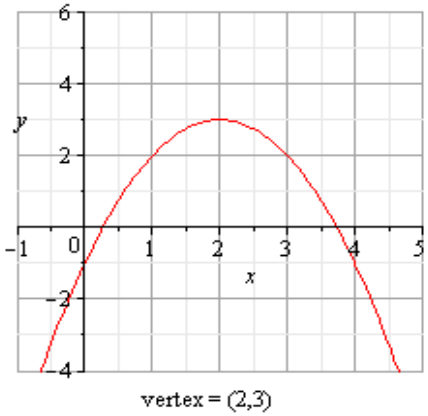
(A) $\frac{1}{2}$ (B) $\frac{5}{7}$ (C) $\frac{6}{7}$ (D) 1 (E) None of these

10. A horticulturist has determined that the number of inches a young oak tree grows in one year is a function of the annual rainfall r given by $g(r) = -0.01r^2 + 0.1r + 2$. What is the maximum number of inches a young oak can grow in one year? The maximum number of inches is:

(A) less than 1 (B) between 1 and 2 (C) between 2 and 3

(D) between 3 and 4 (E) between 4 and 5

11. Find the equation of the parabola whose graph is shown below. The coefficient of x^2 is a number:



(A) between -3 and -1.5 (B) between -1.5 and 0

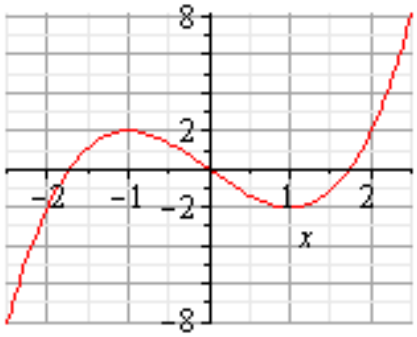
(C) between 0 and 1.5 (D) between 1.5 and 3

(E) None of these

12. Which of the following functions is/are neither even nor odd?

(1) $f(x) = |23x|$ (2) $g(p) = \dfrac{5p}{p^2 + 2}$ (3) $h(t) = 5 - 4t - 2t^3$

(4) $y = f(x)$



(A) All of them (B) 2 and 3 only (C) 2 only

(D) 3 only (E) None of them

13. Given $f(x) = 6x + 3$ and $g(x) = \sqrt{x}$, find $(f \circ g)(x)$.

(A) $\sqrt{6x+3}$ (B) $6\sqrt{x} + 3$ (C) $\sqrt{3} + \sqrt{6x}$

(D) $6x\sqrt{x} + 3\sqrt{x}$ (E) None of these

14. Find $Q^{-1}(t)$ if $Q(t) = \dfrac{C}{4t - 1}$. (C is a nonzero real number)

(A) $Q^{-1}(t) = \dfrac{C}{4}t + C$ (B) $Q^{-1}(t) = \dfrac{4t-1}{C}$

(C) $Q^{-1}(t) = \dfrac{C+t}{4t}, t \neq 0$ (D) $Q^{-1}(t) = \dfrac{C-4t}{t}, t \neq 0$

(E) None of these

15. Suppose $g(4) = 30$ means the volume of water in a container is 30 ounces when the depth of the water is 4 inches. What is the meaning of $g^{-1}(50) = 10$?

(A) The volume of the water is 10 ounces when the depth of the water is 50 inches.

(B) The depth of the water is 10 inches when the volume of the water is 50 ounces.

(C) The depth of the water is 0.2 inches when the volume of the water is 50 ounces.

(D) The volume of the water is 5 ounces when the depth of the water is 10 inches.

(E) None of these

16. If $f(x)$ is a one-to-one function, and $f(8) = 11,$ then which of the following CANNOT be true?

(A) $f(11) = 8$ (B) $f^{-1}(11) = 8$ (C) $f^{-1}(5) = 3$

(D) $f^{-1}(11) = 5$ (E) $f(-8) = -11$

17. Find all the real zeros of $f(x) = x^3 + 5x^2 + 7x + 2$. The largest real zero is:

(A) $\frac{-3+\sqrt{5}}{2}$ (B) -0.5 (C) $\frac{-3+\sqrt{13}}{2}$

(D) $\frac{-3+\sqrt{7}}{2}$ (E) -2

18. Which of the following MUST be true?

(1) A polynomial of degree 4 has 4 unique zeros.

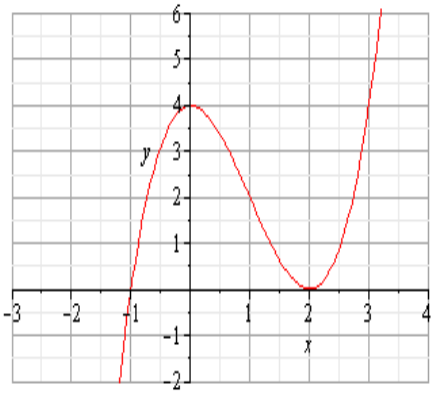
(2) A polynomial of degree 5 has at least 1 real zero.

(3) A polynomial of degree 2 has at least 1 real zero.

(A) 1 only (B) 2 only (C) 3 only

(D) 1 and 2 only (E) 1 and 3 only

19. Which of the following could be the equation of the polynomial $P(x)$ graphed below?



(A) $P(x) = (x+2)^2(x-1)$ (B) $P(x) = 2(x-2)^2(x+1)$

(C) $P(x) = (x-2)^2(x+1)$ (D) $P(x) = -2(x-2)^2(x+1)$

(E) None of these

20. Which of the following statements is/are equivalent to "$x+5$ is a factor of the polynomial $f(x)$"?

(1) $x=5$ is a solution to $f(x)=0$.

(2) $x=-5$ is a zero of $f(x)$.

(3) $(-5,0)$ is an x-intercept of $f(x)$.

(A) 1 only (B) 2 only (C) 3 only

(D) 1 and 3 only (E) 2 and 3 only

21. What is the remainder when $p(x) = x^4 + x^3 - x^2 - 2$ is divided by $x-3$?

(A) -26 (B) -17 (C) 0 (D) 43 (E) None of these

22. Find all the rational zeros of $p(x) = x^4 - 4x^3 + 2x^2 + 5x - 2$. The SUM of all the RATIONAL zeros is:

(A) 1 (B) 2 (C) -1 (D) 0 (E) There are no rational zeros.

23. Find the vertical asymptote(s), if any, for $f(x) = \dfrac{x-2}{3x^2 - 5x}$.

(A) $x = 2$ (B) $x = 0$ (C) $x = 0$ and $x = \dfrac{5}{3}$

(D) $x = \dfrac{5}{3}$ (E) There are no vertical asymptotes.

24. Which of the following statements is/are true about $g(t) = \dfrac{-t^2 + 6}{4t^2 - 7t + 2}$?

(1) $g(t) \to -\dfrac{1}{4}$ as $t \to \infty$

(2) $g(t)$ has a slant asymptote.

(3) 0 is in the range of $g(t)$

(A) 1 only (B) 2 only (C) 3 only

(D) 1 and 2 only (E) 1 and 3 only

25. Let $f(x) = C \cdot b^x$. Determine constants C and b so that $f(-1) = 10$ and $f(2) = \dfrac{5}{4}$.

(A) $C = 5,\ b = \dfrac{1}{2}$ (B) $C = 10,\ b = \dfrac{1}{4}$ (C) $C = 2,\ b = 5$

(D) $C = 1,\ b = \dfrac{5}{4}$ (E) None of these

26. Which ONE of the following is true about the graph of $y = 5000\,e^{-0.0002x} - 9000$?

(A) $y \to \infty$ as $x \to -\infty$ and $y \to 9000$ as $x \to \infty$.

(B) $y \to \infty$ as $x \to -\infty$ and $y \to -9000$ as $x \to \infty$.

(C) $y \to 5000$ as $x \to -\infty$ and $y \to -9000$ as $x \to \infty$.

(D) $y \to 0$ as $x \to -\infty$ and $y \to \infty$ as $x \to \infty$.

(E) $y \to 5000$ as $x \to -\infty$ and $y \to 0$ as $x \to \infty$.

27. The domain of $y = \log_7(4 - 3x)$ is:

(A) $(\frac{4}{3}, \infty)$ (B) $(0, \infty)$ (C) $(-\infty, \frac{4}{3})$

(D) $[\frac{4}{3}, \infty)$ (E) $(-\infty, \infty)$

28. Which of the following statements is/are true about $f(x) = \log_b(x)$? $(b > 0, b \neq 1)$

(1) The domain is $(0, \infty)$

(2) $(0,1)$ is the y-intercept

(3) The range is $(-\infty, \infty)$

(4) The x-intercept is $(b,0)$

(A) 2 and 3 only (B) 1 and 3 only (C) 2, 3, and 4 only

(D) 1, 2, and 4 only (E) 1 only

29. Find the x-intercept of the graph of $y = \ln(x-k)+2$.

(A) $(e^{-2}+k,0)$ (B) $(\ln(-k)+2,0)$ (C) $(e+k,0)$

(D) $(e^{k-2},0)$ (E) None of these

30. Express as a single logarithm and simplify if possible:

$$\frac{1}{3}\log x + 4\log y - 2\log z$$

(A) $\log(\frac{1}{3}x+4y-2z)$ (B) $\log(x^{\frac{1}{3}}+y^4+z^3)$ (C) $\frac{7}{3}\log(\frac{xy}{z})$

(D) $\log(\frac{x^{\frac{1}{3}}y^4}{z^2})$ (E) None of these

31. Solve for x: $3^x = 5^{x-1}$. The solution is a number:

(A) between 2 and 4 (B) between -5 and -3

(C) between -1 and 0 (D) between -3 and -1

(E) None of these

32. How much MORE money will you earn in an account that compounds interest continuously than in an account compounds interest quarterly if you invest $3000 for 7 years at an interest rate of 11%?

(A) $67.02 (B) $59.37 (C) $101.16 (D) $32.52 (E) None of these

33. In 1980, the population of the United States was approximately 226.5 million people. In 1990, the population had grown to approximately 246.7 million people. Assuming an exponential growth model $n(t) = n_0 e^{rt}$, what is the projected population of the U. S. in the year 2000?

(A) Less than 260 million people.

(B) Between 260 and 265 million people.

(C) Between 265 and 270 million people.

(D) Between 270 and 275 million people.

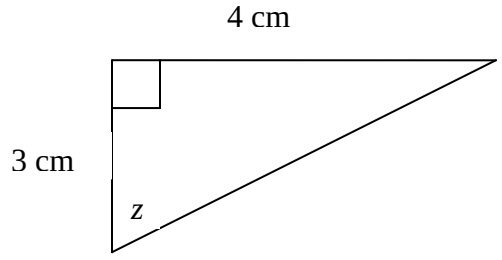
(E) More than 275 million people.

34. The release of fluorocarbons used in household sprays destroys the ozone layer in the upper atmosphere. Suppose the amount of ozone is given by $m = m_0 e^{-0.0025t}$, where t is measured in years. How long will it take for 70% of the ozone to disappear, rounded to the nearest year?

(A) 143 years (B) 1699 years (C) 1360 years

(D) 482 years (E) None of these

35. Find z as a degree measure rounded to one decimal place.



(A) 41.4^o (B) 48.6^o (C) 36.9^o (D) 53.1^o (E) None of these

36. Find the period of $y = -4\cot(2x)$

(A) π (B) $\frac{\pi}{2}$ (C) 2π (D) $\frac{\pi}{4}$ (E) None of these

37. Simplify the expression $\frac{1}{\cos^2\theta} - 1$ completely. The result is

(A) $\cot^2\theta$ (B) $\sec^2\theta$ (C) 0 (D) $\tan^2\theta$ (E) None of these

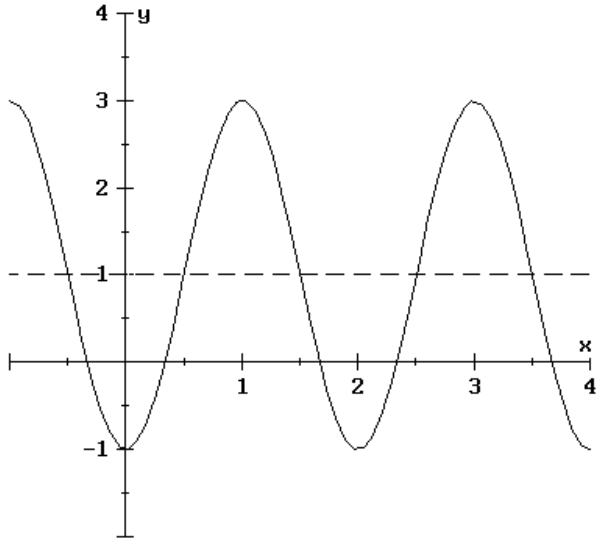
38. Simplify $\frac{\sec x}{\csc x}$ completely.

(A) 1 (B) $\tan x$ (C) $\tan^3 x$ (D) $\cot x$ (E) $\cot^2 x$

39. If the radius of a circle is 3 cm., then the measure of an angle that cuts an arc of length 6 cm is:

(A) 120° (B) π (C) 2 (D) 30^o (E) 0.5

40. The equation of the graph below is

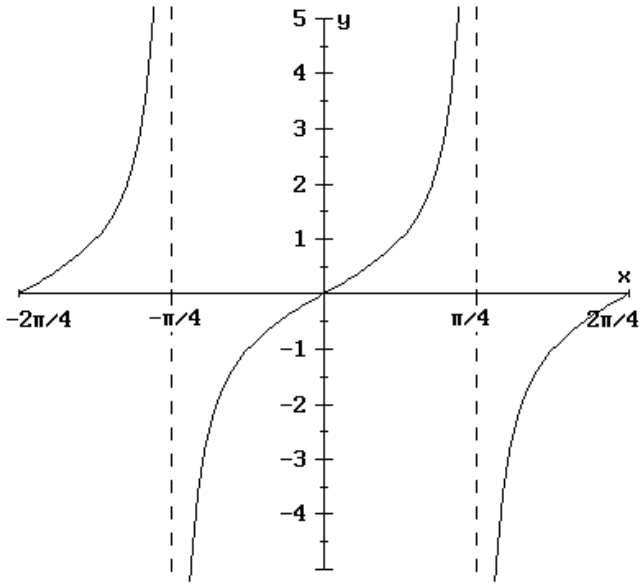


(A) $f(x) = 2 + 3\cos(2x + 2)$ (B) $f(x) = 1 + 2\cos(\pi x + \pi)$

(C) $f(x) = 1 + 2\sin(\pi x - \pi)$ (D) $f(x) = 2 + 3\sin(2x - 2)$

(E) None of these

41. Find the equation of the following graph.



(A) $y = \tan\frac{\pi}{2}x$ (B) $y = \tan 4x$ (C) $y = \tan 2x$

(D) $y = \tan\frac{1}{2}x$ (E) None of these

42. Suppose the angle A terminates in Quadrant III and $\sin A = -\frac{3}{5}$. Find $\cos A$.

(A) $-\frac{4}{5}$ (B) $\frac{4}{5}$ (C) $\frac{3}{5}$ (D) $-\frac{3}{5}$ (E) None of these

43. Suppose the angle A terminates in Quadrant II and $\sin A = \frac{3}{5}$. Find $\tan A$.

(A) $\frac{3}{4}$ (B) $-\frac{3}{4}$ (C) $\frac{4}{3}$ (D) $-\frac{4}{3}$ (E) None of these

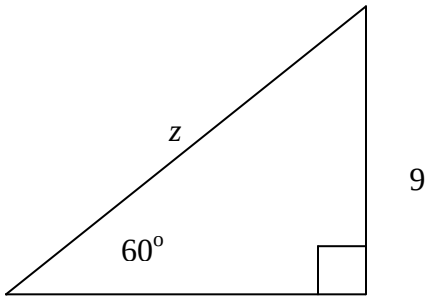
44. Find $\cos\left(\frac{7\pi}{6}\right)$ exactly.

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{-\sqrt{3}}{2}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) None of these

45. Find $\sec\left(\frac{-4\pi}{3}\right)$ exactly.

(A) 2 (B) $\frac{2}{\sqrt{3}}$ (C) $-\frac{2}{\sqrt{3}}$ (D) $-\frac{1}{2}$ (E) None of these

46. Find *z* exactly.



(A) 9 (B) $\frac{9\sqrt{3}}{2}$ (C) $\frac{18}{\sqrt{3}}$ (D) 18 (E) None of these

47. Suppose $x = n$ with $0 \le n \le \frac{\pi}{2}$ is a solution to the equation $\sin x = m$. Find another solution to the equation $\sin x = m$.

(A) $-n$ (B) $2\pi - n$ (C) $\pi - n$ (D) $\pi + n$ (E) None of these

For problems 48 and 49, consider the function $f(t) = -5\sin 2(t+4) + 6$.

48. The amplitude is:

(A) –5 (B) –4 (C) 2 (D) 5 (E) None of these

49. The maximum value is:

(A) 1 (B) 4 (C) 6 (D) 11 (E) None of these

50. If cos $m = n$ and sin $m = k$, then

(A) $\cos(-m) = -n$ and $\sin(-m) = -k$ (B) $\cos(-m) = n$ and $\sin(-m) = k$

(C) $\cos(-m) = -n$ and $\sin(-m) = k$ (D) $\cos(-m) = n$ and $\sin(-m) = -k$

(E) None of these

51. On what interval is the identity $\sin^{-1}(\sin x) = x$ valid?

(A) $[0, 2\pi]$ (B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (C) $[0, \pi]$ (D) $[-1,1]$ (E) None of these

52. What is the domain of $y = \tan^{-1} x$?

(A) $(-\infty,\infty)$ (B) $[0, 2\pi]$ (C) $[0, \pi]$ (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (E) None of these

53. How many distinct solutions does the equation $\sin(3x) = \frac{1}{2}$ have on the interval $[0,2\pi)$?

(A) 1 (B) 3 (C) 6 (D) 12 (E) Infinitely many

Short Response Practice

54. The relationship between the tuition in dollars, *T*, and the number of credits, *c*, at a particular college is given by

$$T(c)=\begin{cases}100+120c & 0\le c\le 6\\ 800+120(c-6) & 6<c\le 18\end{cases}$$

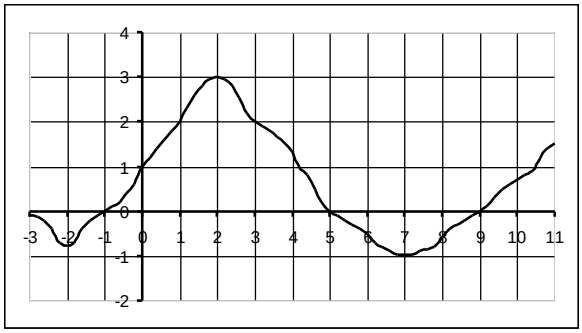
a. What is the tuition for 7 credits?

b. If the tuition was \$1880, how many credits were taken?

c. What is the domain of this function?

d. What are the practical interpretations of the y - intercept and the slope?

55. Use the graph at the right to answer the questions below.



a. Find $f(0)$.

b. On what intervals is $f(x)$ increasing?

c. Find x so that $f(x)=2$.

d. Find the zeros of $f(x)$.

e. What is $f(f(9))$?

56. Let $f(t)=-t^2+7t-5$. Find and simplify $\dfrac{f(t+h)-f(t)}{h}$.

57. Suppose the tax bill $T(y)$ of a single person whose adjusted gross income is y dollars is represented by a linear model. Assume the tax bill for a person with \$9550 in adjusted gross income is \$1015, and the tax bill for a person with \$19650 in adjusted gross income is \$2530.

a. Determine the formula for $T(y)$.

b. Explain what the average rate of change of T tells you in terms of the tax bill and adjusted gross income.

58. A piece of wire 20 cm long is cut into two pieces. The first is bent into a circle, the second into a square. Express the combined total area of the circle and square as a function of x, where x represents the length of the wire that is bent into a circle.

59. The volume of a right circular cone is given by the formula $V = \frac{1}{3}\pi r^2 h$. If the volume of a right circular cone is 50 cubic centimeters, express the radius as a function of height.

60. An airplane manufacturer can produce up to 15 planes per month. The profit made from the sale of these planes can be modeled by $P(x) = -0.2x^2 + 4x - 3$ where $P(x)$ is the profit in hundred thousand of dollars per month and x is the number of planes made and sold. Based on this model, how many planes should be made and sold to maximize the profit? What is the maximum profit?

61. A farmer introduces 100 trout into his pond. The population of the trout can be modeled by the function $p(t) = \dfrac{150t + 100}{0.04t + 1}$, where t is time measured in months. Find and give an interpretation of the horizontal asymptote of $p(t)$.

62. A spherical balloon is being inflated. Suppose the radius of the balloon is increasing at a rate of 2 centimeters per second.

    a. Express the radius r of the balloon as a function of time t.

    b. Express the volume V of the balloon as a function of time t.

63. The formula $K = \dfrac{F - 32}{1.8} + 273.15$ is used to determine the corresponding Kelvin measure K of a temperature with Farhenheit measure F. Find and simplify the inverse of K. Give a practical interpretation of the inverse of K.

64. Determine the shortest distance from the ordered pair $(1,4)$ to the graph of $y = x^2$. Approximate your answer to the nearest hundredth.

65. Suppose $\cos t = \dfrac{2}{3}$ and that the terminal point of t is in Quadrant IV. Use the fundamental identities to find the exact value of $\sin t, \csc t, \text{and} \tan t$.

66. Solve for the indicated variable. Give exact answers.

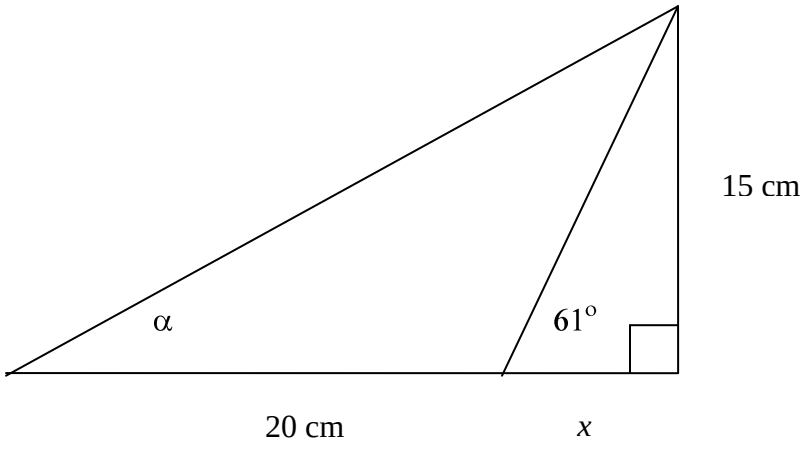
a. $\log(y-1)+\log(y+1)=\log 8$

b. $e^{(5x-2)}+9=13$

c. $2\sin\theta\cos\theta=\dfrac{\sqrt{3}}{2}$ on the interval $[0,2\pi)$

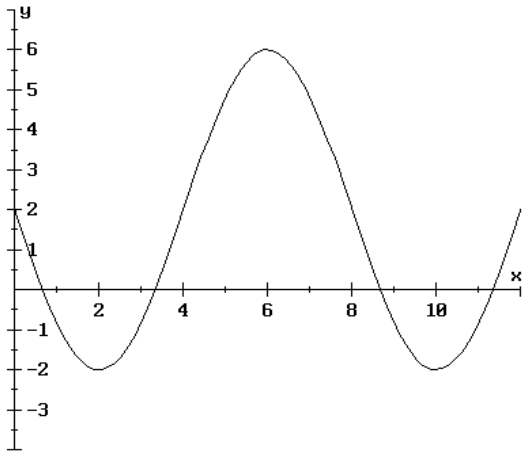
d. $3\sin^2 t=2\sin t+1$ on the interval $[0,2\pi)$

67. Find α and x. Approximate your answers to the nearest hundredth.

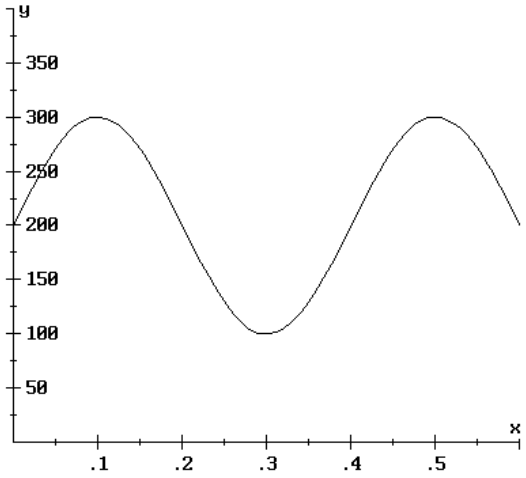


68. Find a possible formula for each graph

a.



b.



69. Simplify each expression. Your final answer should be exact and should not contain any trigonometric expression.

a. $\cos(\tan^{-1}(2))$

b. $\sin(\cos^{-1}(x))$

70. The top floor of a building undergoes damped harmonic motion after a sudden brief earthquake. At time $t = 0$, the displacement is at a maximum, 16 centimeters from the normal position. The damping constant is $c = 0.72$ and the building vibrates at 1.4 cycles per second.

a. Find a function of the form $y = ke^{-ct}\cos\omega t$ to model the motion.

b. What is the displacement after 10 seconds?