Exact values of quantum violations in low-dimensional Bell correlation inequalities

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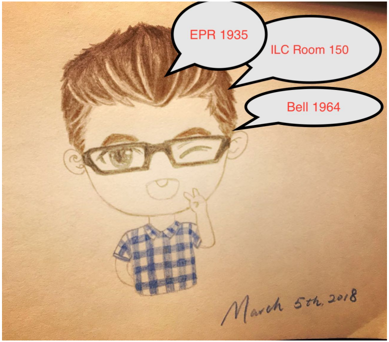
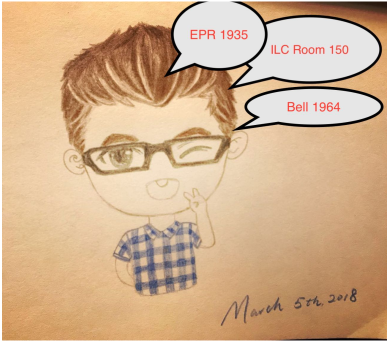

EPR 1935
ILC Room 150
Bell 1964
March 5th, 2018



How much different are quantum and classical descriptions of reality?

Definition

A $m \times n$ real matrix (a_{ij}) is called a classical correlation matrix if there exist random variables $(X_i)_{1 \leq i \leq m}$, $(Y_j)_{q \leq j \leq n}$ defined on a common probability space, satisfying $|X_i| \leq 1, |Y_j| \leq 1$ almost surely, and such that $a_{ij} = \mathbb{E}X_i Y_j$ for all i, j. The set of classical correlation matrices is denoted by $\mathbf{LC}_{m,n}$.

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Definition

A $m \times n$ real matrix (a_{ij}) is called a quantum correlation matrix if there is a quantum state ρ on $\mathbb{C}^{d_1} \otimes \mathbb{C}^{d_2}$) (for some $d_1, d_2 \in \mathbb{N}$), Hermitian operators $(X_i)_{1\leq i\leq m}$ on $\mathbb{C}^{d_1}$, $(Y_j)_{q\leq j\leq n}$ on $\mathbb{C}^{d_2}$ satisfying $\|X_i\| \leq 1, \|Y_j\| \leq 1$ and such that $a_{ij} = \mathrm{Tr}\rho(X_i \otimes Y_j)$ for all i, j. The set of quantum correlation matrices is denoted by $\mathbf{QC}_{m,n}$.

Remark

$\mathbf{LC}_{m,n}$ *and* $\mathbf{QC}_{m,n}$ *have very simple geometric descriptions.*

Theorem (Grothendieck-Tsirelson Theorem, Grothendieck 50s' , Tsirelson85)

There exist $K \geq 1$ and positive integers m, n, the following two conditions are equivalent:
1. We have the inclusion

$$\mathbf{QC}_{m,n} \subset K\,\mathbf{LC}_{m,n}.$$

2. For any $m \times n$ real matrix $(m_{i,j})$ and for any real Hilbert space vectors x_i, y_j with $|x_i| \leq 1, |y_j| \leq 1$ we have

$$\sum_{i,j} m_{i,j}\langle x_i, y_j\rangle \leq K \max_{\xi_i=\pm 1, \eta_j=\pm 1} \sum_{i,j} m_{i,j}\xi_i\eta_j. \tag{1}$$

Moreover, there exists an absolute constant $K \geq 1$ such that 1. and 2. hold for any positive integers m, n.

Clauser-Horne-Shimony-Holt inequality (CHSH): for d=2,

$$\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

For d=4,

$$CHSH_4 = \frac{1}{2}\begin{bmatrix}1 & 1 & 0 & 0\\ 1 & -1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

The maximal violation of $CHSH_4$ on $\mathbf{QC}_{4,4}$ is $\sqrt{2} \approx 1.4142$.

Two new Bell correlation inequalities appear (Avis, Imai, Ito 2006)

$$4_1 = \frac{1}{6}\begin{bmatrix} 2 & 1 & -1 & 0 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \quad 4_2 = \frac{1}{10}\begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 1 & -2 \\ 2 & 1 & -2 & 1 \\ 2 & -2 & 1 & -1 \end{bmatrix}$$

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The maximal violation of 4_1 on $\mathbf{QC}_{4,4}$ is $\frac{5}{3}\sqrt{\frac{2}{3}} \approx 1.3608$.

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The maximal violation of 4_2 on $\mathbf{QC}_{4,4}$ is $\frac{2}{5}\sqrt{10+\sqrt{2}} \approx 1.3514$.

Ref: B. L, arXiv 1712.08506.

Thank You!