Approximating the Variance of Coefficients for the Characteristic Polynomial of Binary Quantum Graphs

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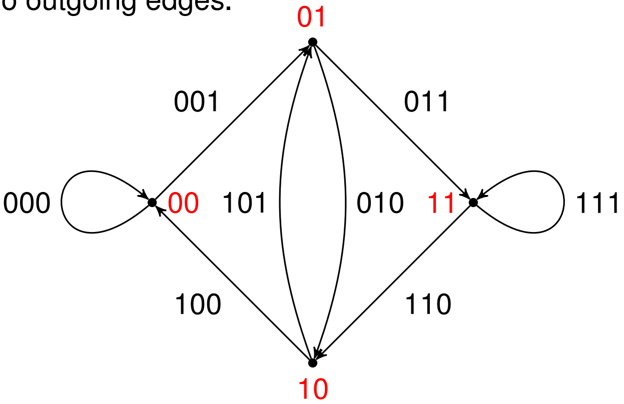
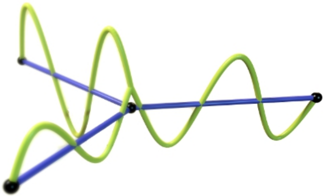
Binary Graphs

- $V = 2^r$
- $E = 2V = 2^{r+1}$
- G a directed graph ($B = E$) where each vertex has two incoming and two outgoing edges.



Quantum Graphs



- We study wave functions, $\psi = c_1 e^{ikx} + c_2 e^{-ikx}$ on graph, G.
- Let the *scattering matrix* be the *Discrete Fourier Transform* matrix:

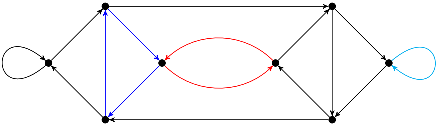
$$\sigma^{(v)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

- Collect entries into a *bond scattering matrix*:

$$S_{b',b} := \delta_{t(b),v}\, \delta_{o(b'),v}\, \sigma^{(v)}_{b',b}$$

- *Quantum evolution operator* is $U = Se^{ikL}$ for $L = \mathrm{diag}(L_1, \ldots, L_B)$.

Periodic & Pseudo Orbits



- A *periodic orbit*, $\gamma = (e_1, \ldots, e_m)$, is an equivalence class of closed paths under rotation.
- A *pseudo orbit*, $\tilde{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_M)$ is a collection of periodic orbits.
- We use orbits to investigate spectral statistics of G.

Characteristic Polynomial

- The *characteristic polynomial* of a matrix, $U(k)$, is given by:

$$F_\xi(k) = \det(\xi \mathbf{I} - U(k)) = \sum_{n=0}^{B} a_n(k)\xi^{B-n}$$

- For $F_1(k) = 0$ with $U(k)$ the quantum evolution operator, we have the *secular equation* of G, which determines the spectrum of G.

Theorem (R. Band, J. M. Harrison, C. H. Joyner; 2012)

The coefficients of the characteristic polynomial $F_\xi(k)$ can be computed by sums over pseudo orbits.

- We compute the variance of these coefficients by sums over pairs of pseudo orbits.

Approximations of the Variance

Theorem (R. Band, J. M. Harrison, M. Sepanski; 2016)

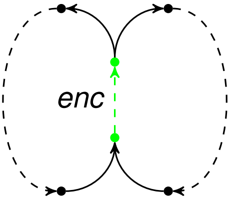
Pairing a pseudo orbit with itself contributes $\langle |a_n|^2 \rangle_{\mathrm{diag}} = 1/2$.

Proof idea: Each pair contributes a term 2^{-n}, and the number of pairs of the appropriate type is 2^{n-1}.

Theorem (J. M. Harrison, H.; 2017)

Pairs containing a “figure eight” *encounter* contribute $\langle |a_n|^2 \rangle_{\mathrm{fig8}} = 0$.

Proof idea: Encounter length determines whether pair contributes $\pm 2^{-(n+L_{enc})}$ to sum. Take limit of graph size, pseudo orbit length, and encounter length to infinity, and sum tends to zero.



Questions?

Thank You