Landauer's principle for trajectories of repeated interaction systems

Renaud Raquépas,
Université Grenoble Alpes, McGill University

joint work with
E. P. Hanson, A. Joye and Y. Pautrat

Arizona School of Analysis and Mathematical Physics
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Repeated interaction systems (RIS)

The small system $\mathcal{S}$ is decribed by

a finite dimensional **Hilbert space** $\mathcal{H}_\mathcal{S}$,

a **Hamiltonian** $h_\mathcal{S} \in \mathcal{B}(\mathcal{H}_\mathcal{S})$,

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The environment is composed of probes $\{\mathcal{E}_k\}_{k=1}^{T}$ described by

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Let $\Xi := \bigotimes_{k=1}^{T} \xi_k^{\mathrm{i}}$ on $\bigotimes_{k=1}^{T} \mathcal{H}_{\mathcal{E}}$

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- Evolve jointly with $U_k := \mathrm{e}^{\mathrm{i}\tau(h_{\mathcal{S}} \otimes \mathrm{Id} + \mathrm{Id} \otimes h_{\mathcal{E}_k} + v_k)}$

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- Trace out the probe

$$\mathrm{Tr}_{\mathcal{E}_k}(U_k(\rho_{k-1} \otimes \xi_k^{\mathrm{i}})U_k^*)$$

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the dynamics can be summarized as

$$\begin{aligned} \rho_k &= \mathcal{L}_k(\rho_{k-1}) \\ &= (\mathcal{L}_k \circ \cdots \circ \mathcal{L}_2 \circ \mathcal{L}_1)(\rho^{\mathrm{i}}) \end{aligned}$$

and $\mathcal{L}_k$ is a *completely positive trace-preserving* map (CPTP, quantum channel) for each k.

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[!] For the rest of the talk, there is implicit T-dependence on many quantities.

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The map $\mathcal{L}_k = \mathcal{L}(\frac{k}{T})$ is a CPTP map.

[1] A CP map Λ is irreducible if the only self-adjoint projectors P satisfying $\Lambda(P\mathcal{I}_1 P) \subseteq P\mathcal{I}_1 P$ are $P = 0$ and $P = \mathrm{Id}$.

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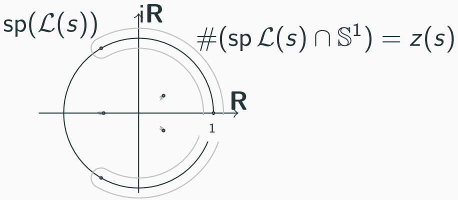

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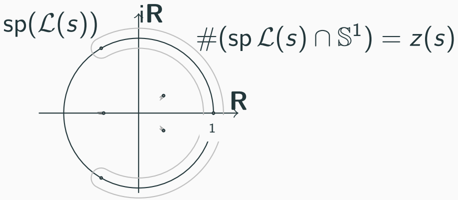

Gap: We assume that $z(s) \equiv z$ is constant in $s \in [0,1]$.

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Two-time measurement protocol

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- Measure the energy observable $h_{\mathcal{E}_k} = \sum_{i_k} E_{i_k} \Pi_{i_k}$ of each probe $\mathcal{E}_k$
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- Let $\mathcal{S}$ interact with each $\mathcal{E}_k$, starting with $k = 1$ until $k = T$, via

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- Measure the energy observable $h_{\mathcal{E}_k} = \sum_{j_k} E_{j_k} \Pi_{j_k}$ of each probe $\mathcal{E}_k$
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Two-time measurement protocol

In this two-time measurement protocol, the probability of obtaining $E_{i_1}, E_{i_2}, \dots, E_{i_T}$ and $-\log p_\ell^{\mathrm{i}}$ before the interaction $(t=0)$ and $E_{j_1}, E_{j_2}, \dots, E_{j_T}$ and $-\log p_\ell^{\mathrm{f}}$ after the interaciton $(t=T)$ is denoted

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We can also write down probability

$$\mathbb{P}_B(\omega)$$

of obtaining these measurement results in a certain notion of backward process.

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- The entropy production random variable

$$\varsigma(\omega) := \log \frac{\mathbb{P}_F(\omega)}{\mathbb{P}_B(\omega)}.$$

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We have the "entropy balance equation" on the level of trajectories

$$\Delta s_{\mathcal{S}}(\omega) + \varsigma(\omega) = \Delta s_{\mathcal{E}}(\omega)$$

for each $\omega = (E_{i_1}, E_{i_2}, \ldots, E_{i_T}, -\log p^{\mathrm{i}}_{\ell}, E_{j_1}, E_{j_2}, \ldots, E_{j_T}, -\log p^{\mathrm{f}}_{\ell})$.

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Moreover

$$\mathbb{E}_F(\Delta s_{\mathcal{S}}) = S(\rho^{\mathrm{f}}) - S(\rho^{\mathrm{i}}),$$

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$$\mathbb{E}_F(\varsigma) = \mathrm{KL}(P_F || P_B) \geq 0.$$

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On the level of averages,

$$\sum_{k=1}^{T} \beta_k \operatorname{Tr}\big(h_{\mathcal{E}_k}(\xi_k^{\mathsf{f}} - \xi_k^{\mathsf{i}})\big) = S(\rho^{\mathsf{f}}) - S(\rho^{\mathsf{i}}) + \mathbb{E}_F(\varsigma)$$
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Under the Gap and Irr assumptions,

$$\lim_{T\to\infty} \mathbb{E}_F(\varsigma) < \infty$$

if and only if

$$\forall s \in [0,1]\ \exists k_{\mathcal{S}}(s) \in \mathcal{B}(\mathcal{H}_{\mathcal{S}}) \text{ s.t. } [k_{\mathcal{S}}(s) \otimes \mathrm{Id} + \mathrm{Id} \otimes h_{\mathcal{E}}(s), U(s)].$$

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What about the details of the distribution of ς with respect to $\mathbb{P}_T$.

Generating function

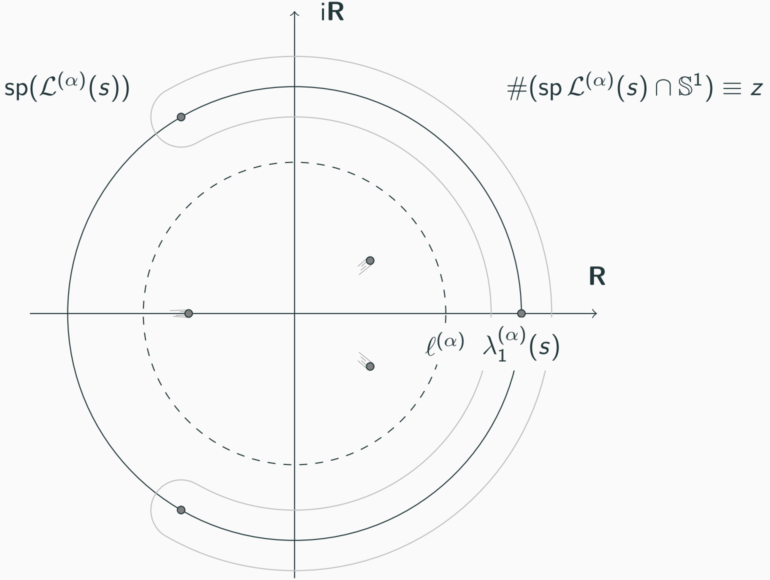
The *moment generating function* of ς is

$$M_\varsigma(\alpha) = \mathsf{Tr}_{\mathcal{S}}\left(\mathrm{e}^{-\alpha \log \rho^{\mathrm{f}}}\mathcal{L}^{(\alpha)}(\tfrac{T}{T})\cdots\mathcal{L}^{(\alpha)}(\tfrac{1}{T})(\mathrm{e}^{\alpha \log \rho^{\mathrm{i}}}\rho^{\mathrm{i}})\right).$$

where

$$\mathcal{L}^{(\alpha)}(s) : \eta \mapsto \mathsf{Tr}_{\mathcal{E}}\left(\mathrm{e}^{\alpha\beta(s)h_{\mathcal{E}}(s)}U(s)(\eta\otimes\xi(s))\mathrm{e}^{-\alpha\beta(s)h_{\mathcal{E}}(s)}U(s)^*\right).$$

An adiabaic theorem



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$$= \Big(\prod_{k=1}^{T} \lambda_1^{(\alpha)}(\tfrac{k}{T})\Big) \mathsf{Tr}_{\mathcal{S}}\left((\rho^{\mathsf{f}})^{-\alpha} \tilde{\mathcal{L}}^{(\alpha)}(\tfrac{T}{T}) \cdots \tilde{\mathcal{L}}^{(\alpha)}(\tfrac{1}{T})((\rho^{\mathsf{i}})^{\alpha+1})\right).$$

Using our adiabatic theorem for deformations deformed CPTP maps,

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By standard techniques (Gärtner–Ellis, Bryc), we get a large deviation principle, a central limit theorem and a law of large number.

Thank you!

If interested in more, see

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