Energy Flows in Electrical Grids

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PROJECT DESCRIPTION

- Develop a model of power and voltage distribution along an electrical line using [1] to understand energy flow in grids
- Adjust model to include consumption variations in loads and analyze effects to overall power and voltage flow

POTENTIAL APPLICATIONS

- Use in smart grid technology to predict voltage fluctuations in a line, allowing for more efficient energy provisions for a given grid
- Explain and prevent undesired events like power outages
- Understand how to transmit energy over long distances, such as from wind turbines
- Better integrate renewable energy sources that input energy into the grid

MODEL FORMULATION

An electrical feed is modeled as a one directional line with a large number of consumers. The real power, reactive power, and voltage along the line can be modeled by the following system of ordinary differential equations with desirable boundary conditions:

$$\frac{d}{dz}\begin{pmatrix}P\\Q\\v\end{pmatrix}=\begin{pmatrix}p-r\frac{P^2+Q^2}{v^2}\\ q-x\frac{P^2+Q^2}{v^2}\\ -\frac{rP+xQ}{v}\end{pmatrix}$$

$$v_0=1, P(L)=Q(L)=0$$

Where:

- z is the position along the line
- p and q represent the real and reactive power consumptions along the line
- r and x represent the resistance and reactance of the line, and
- P and Q represent the real and reactive power flows
- v is the voltage

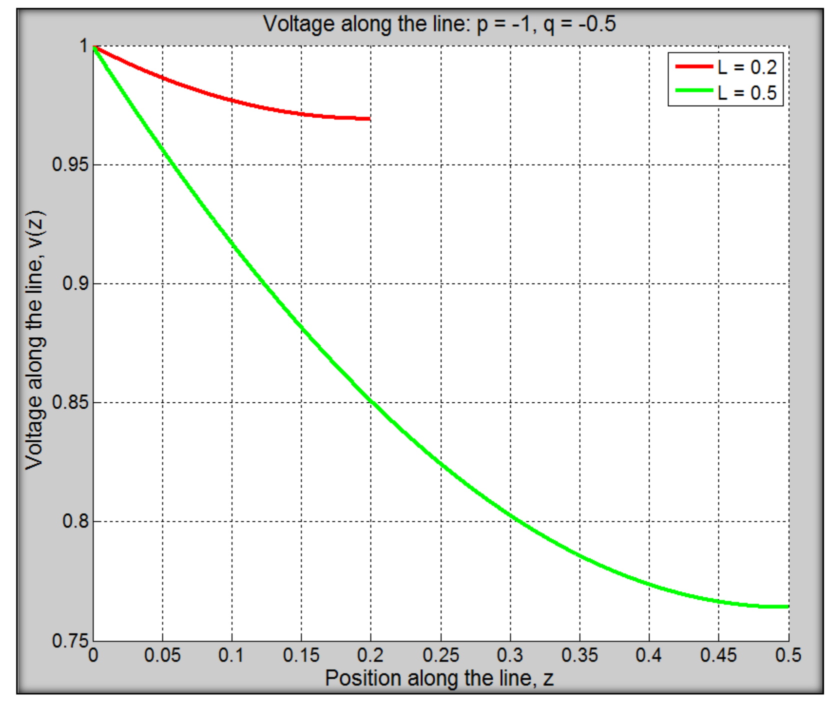
Further, the system can be rescaled to define an initial value problem:

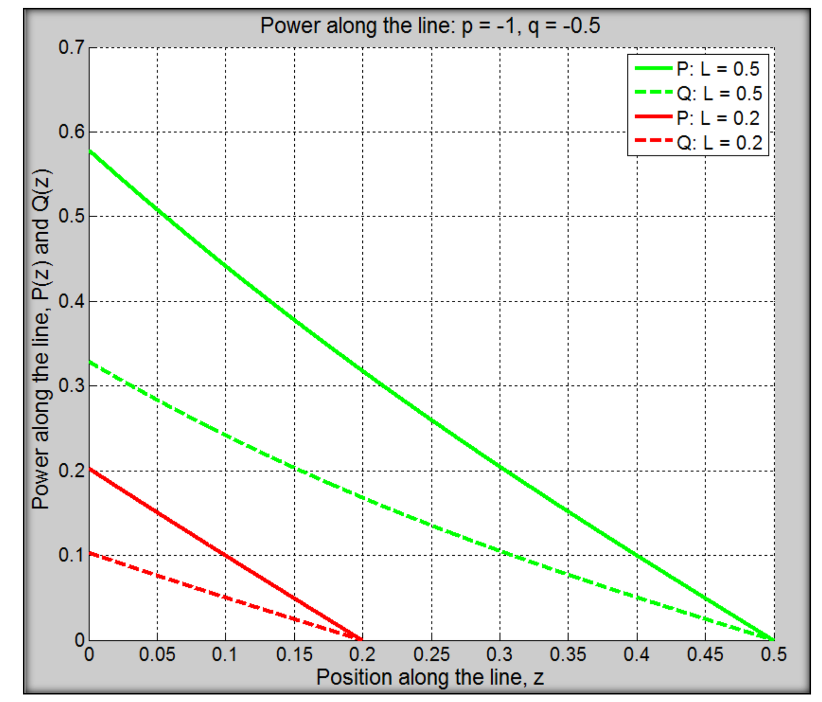
$$\frac{d}{ds}\begin{pmatrix}\varrho\\ \tau\\ v\end{pmatrix}=\begin{pmatrix}sign(p)-\frac{\varrho^2+\tau^2}{v^2}\\ A-B\frac{\varrho^2+\tau^2}{v^2}\\ -\frac{\varrho+B\tau}{v}\end{pmatrix}$$

$$v(0)=1, \varrho(0)=\tau(0)=0$$

RESULTS

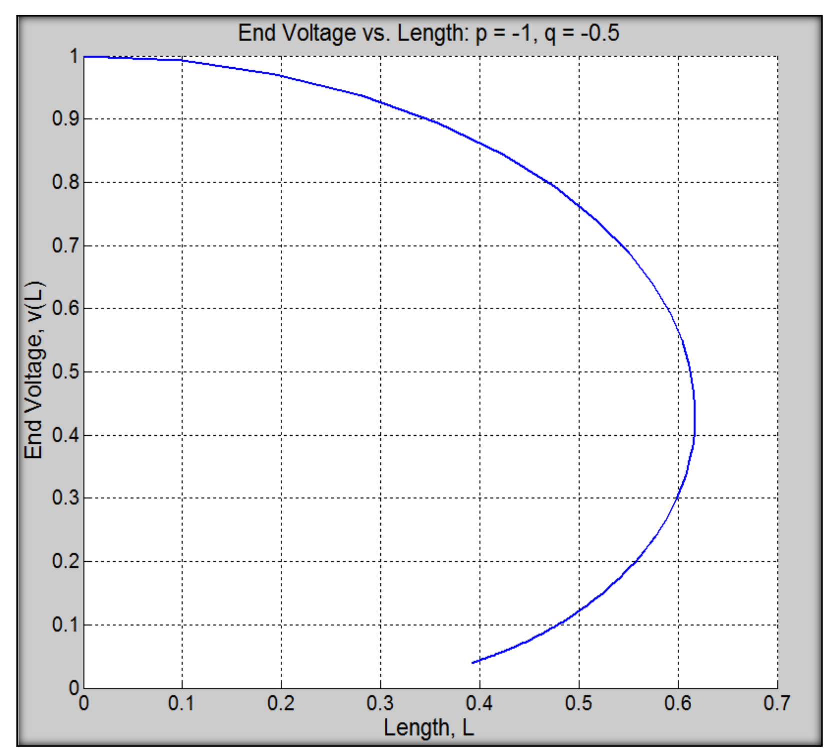
Boundary Value Problem

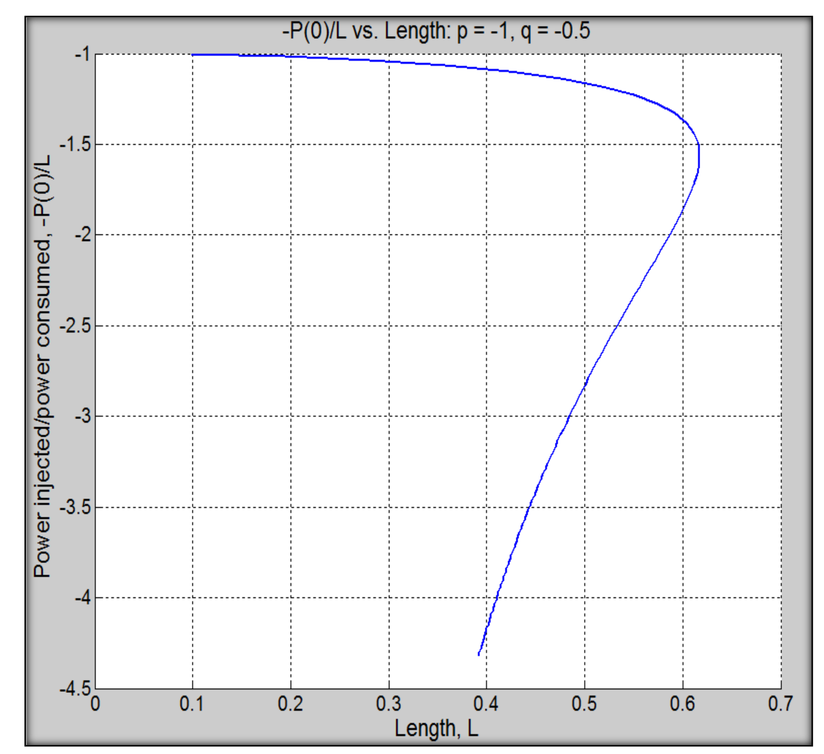




Voltage along the line:
- A negative p leads to power consumption (decreasing curves)
- All initial conditions are met (v(0) = 1, P(0) = Q(0) = 0)

Initial Value Problem





End voltage and power utilization versus length of line:
- There are two stable solutions
  - The higher value, which resulted more often in simulations
  - The lower value, which could lead to power outages
- There is a maximum length of line
  - The length is limited because there is a limit to the amount of power that can be consumed by the system

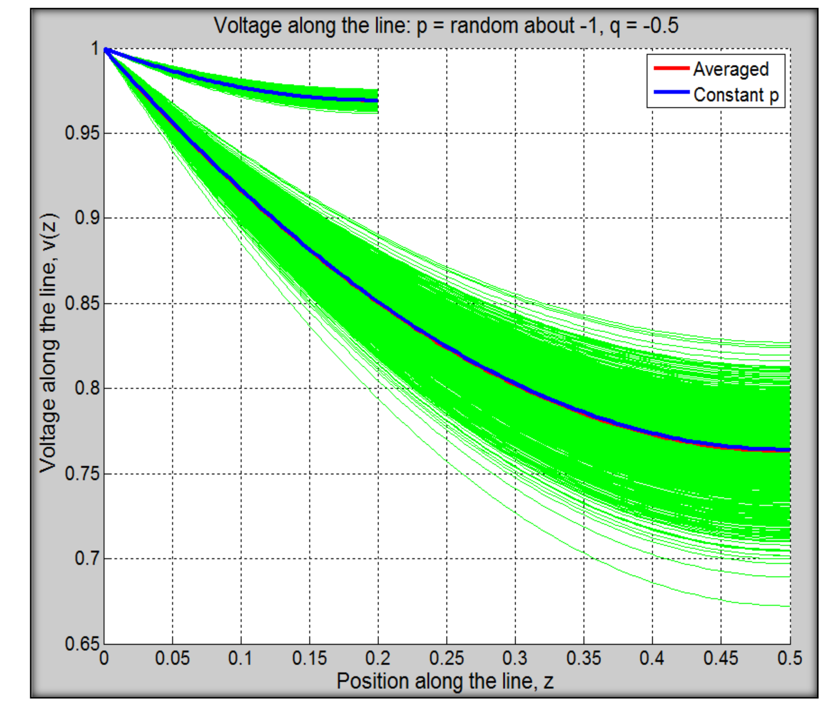
IMPLEMENTATION OF STOCHASTICITY

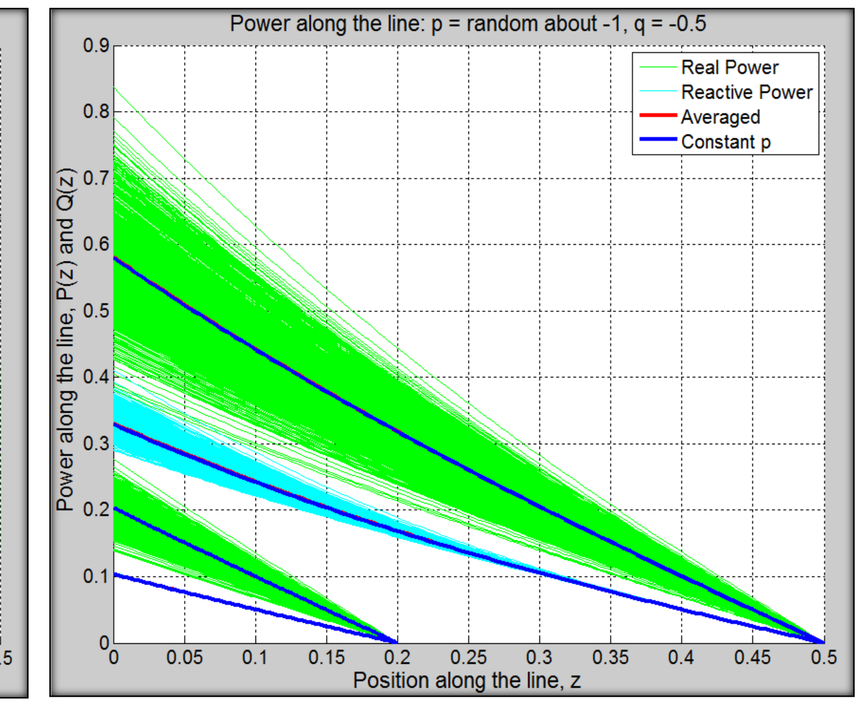
I. A Wiener Process [2] was added to the power consumption, p, in the boundary-value problem, making it a discretely defined function of the position of the line.

II. The boundary-value problem solver was then run with this new power consumption profile, $p(z)$, and this process was repeated 1000 times.

III. Statistics such as mean and higher moments were calculated and analyzed

STOCHASTIC RESULTS





Voltage and power along the line with stochastic element:
- The average of solutions with varying p are almost identical to the solution with constant p
- There was less variation in shorter lines

1. The solutions for the stochastic power consumption tended towards the original solution of constant power consumption.
2. In none of these simulations did the solution jump to the lower branch, indicating a rather stable solution, regardless of the consumption profile.
3. The variance of the end voltages was much less than that of the power consumption profiles.
4. From the simulations, it was seen that a high variation in power consumption along a power line has little influence upon the stability of the line. Therefore, it is statistically valid to assume that there is a constant power consumption across the line.

GLOSSARY OF TECHNICAL TERMS

Real Power: Real component of complex power that is consumed.
Reactive Power: Imaginary component of complex power that is stored and returned to the system

REFERENCES

1. D Wang, K Turitsyn and M Chertkov, "DistFlow ODE: Modeling, Analyzing and Controlling Long Distribution Feeder", Proceedings of, the 51st IEEE Conference on Decision and Control (2012).
2. bpath1.m was developed by Jeffrey Moehles, Department of Mechanical Engineering, University of California, Santa Barbra.

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