Sinking Bubbles in an Oscillating Fluid

Project Description

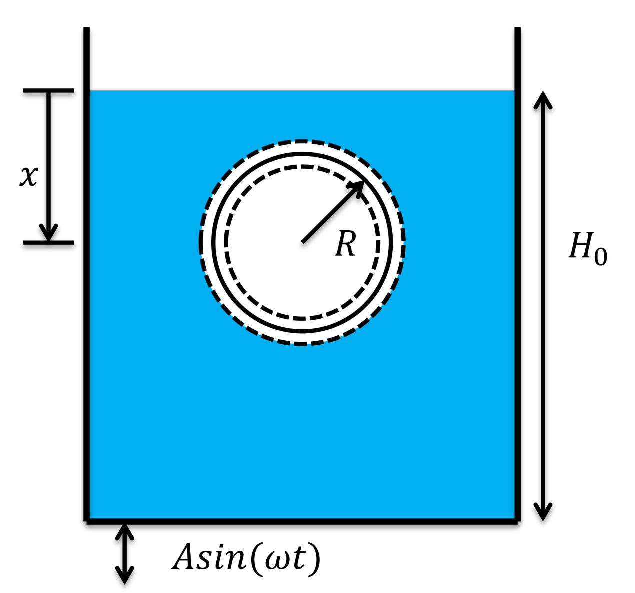
- Air bubble in water can be shown to sink with the laws of fluid mechanics. [1]
- We attempted to model the behavior using direct integration by MATLAB code and direct separation of motions for computational and analytical solutions respectively. [2]
- Compared Results of analytical and direct integration solutions.
- Created bifurcation diagram showing changes in bubble behavior based on variable parameter choices

Scientific Challenges

- Bubble and fluid motions are highly coupled and nonlinear:
- Multiple competing forces on bubble: gravity, buoyancy, oscillations, drag, and added (attached) mass.

Potential Applications

- Bubble sinking caused an unexpected accumulation of air at the bottom of rocket fuel tanks. Prevented early success of 3-stage rockets.
- Accurate model required for success of rockets as well as advanced liquid cooling technologies. [3]
- Practical application for separation of motions (slow and fast time scales)



- Variables and parameters
  - x – Bubble depth
  - R – Bubble radius
  - H_0 – Total height of water
  - A – Amplitude of oscillations
  - ω – Frequency of oscillations
  - t – Time

Schematic Diagram of our model. Parameters of x_0 (initial depth) and ω varied in model. A and R_0 (bubble radius near free surface of the fluid) chosen to work with physical assumptions in model.

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Methodology

1. Using fluid mechanics and physical approximations (quasi-static, no cavitation, turbulent flow) we developed an equation of motion for a spherical bubble:

$$(m+m_0)\ddot{x} + \dot{m}_0\dot{x} = -F(\dot{x}) + (\mathrm{m} - \rho V_b)(A\omega^2 \sin\omega t + \mathrm{g})$$

$$m_0 = \frac{1}{2}\rho V_b$$
$$F(\dot{x}) = \frac{1}{2}\rho \dot{x}^2 C_d A * \operatorname{sign}(\dot{x})$$
$$V_b = V_{b0}\left[1 - \frac{\rho x g}{P_e}\left(1 + \frac{A\omega^2}{g}\sin\omega t\right)\right]$$

m = bubble mass
ρ= density of water
V_b= volume of bubble
C_d = drag coefficient of bubble
A= surface area of bubble
g= gravitational acceleration
P_e = atmospheric pressure

2. Separation of motions [2]: $x(t,\tau) = X(t) + \Psi(t,\tau)$

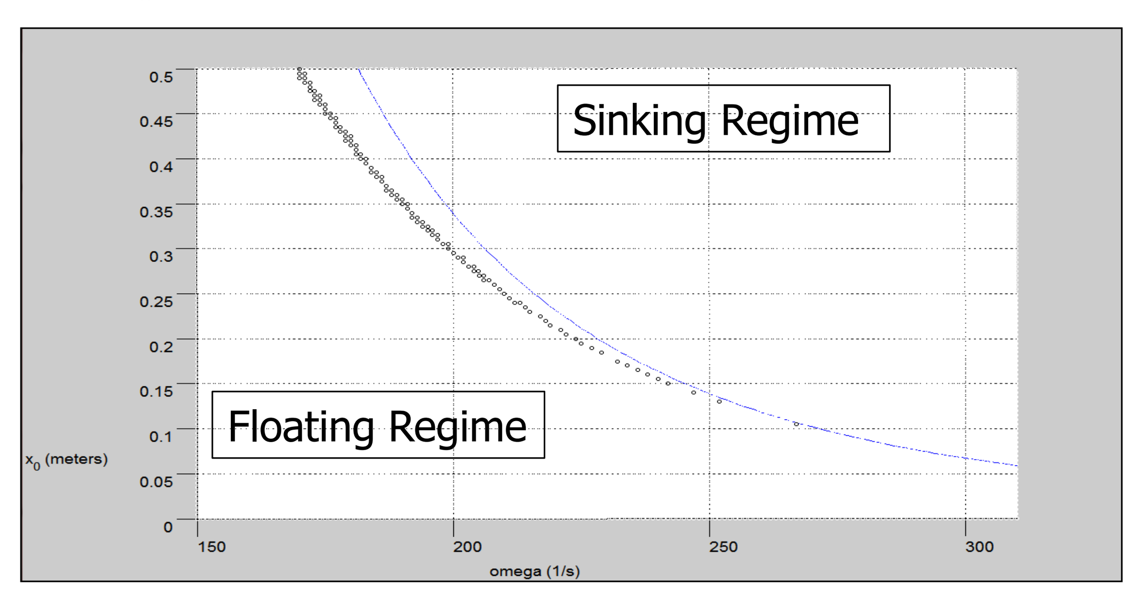
   Fast motion of bubble: $\langle \Psi(t,\tau) \rangle = 0$

   Slow motion of bubble: $\langle X(t) \rangle = X(t)$

   $\langle x(t,\tau) \rangle = X(t)$

Results

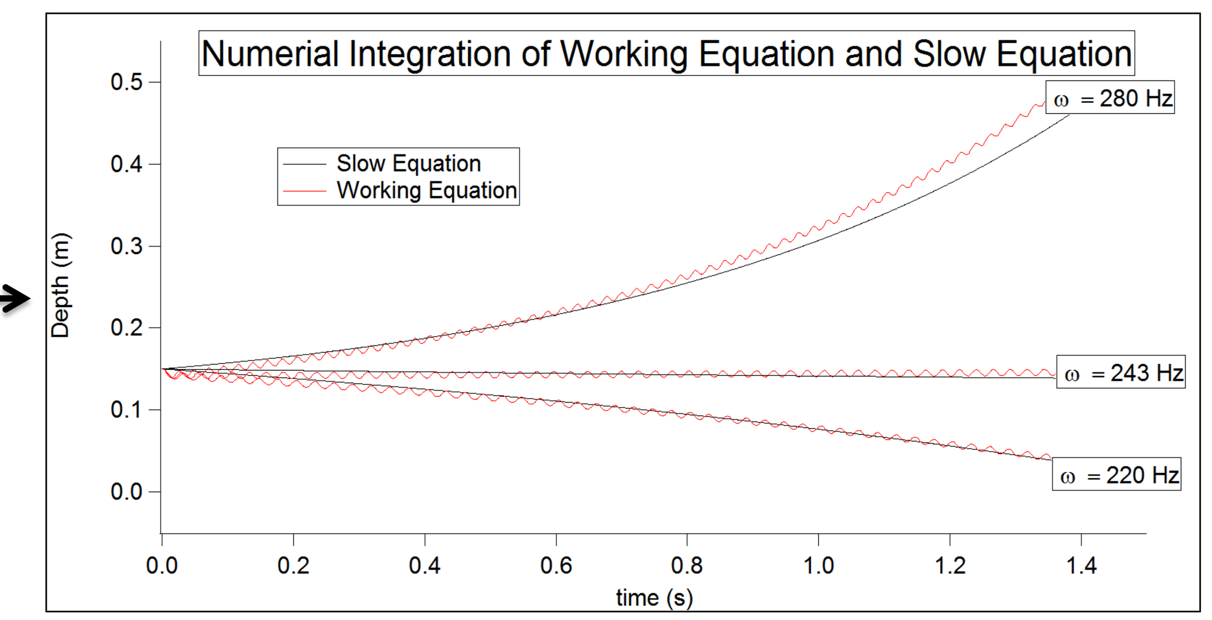
1. Three distinct bubble behaviors found from direct integration and analytical solutions: floating, sinking, and unstable trapping
2. Stability and Instability Regimes mapped out as a function of variable parameters.
3. Prediction of instability from analytical solution also compared to results from direct integration



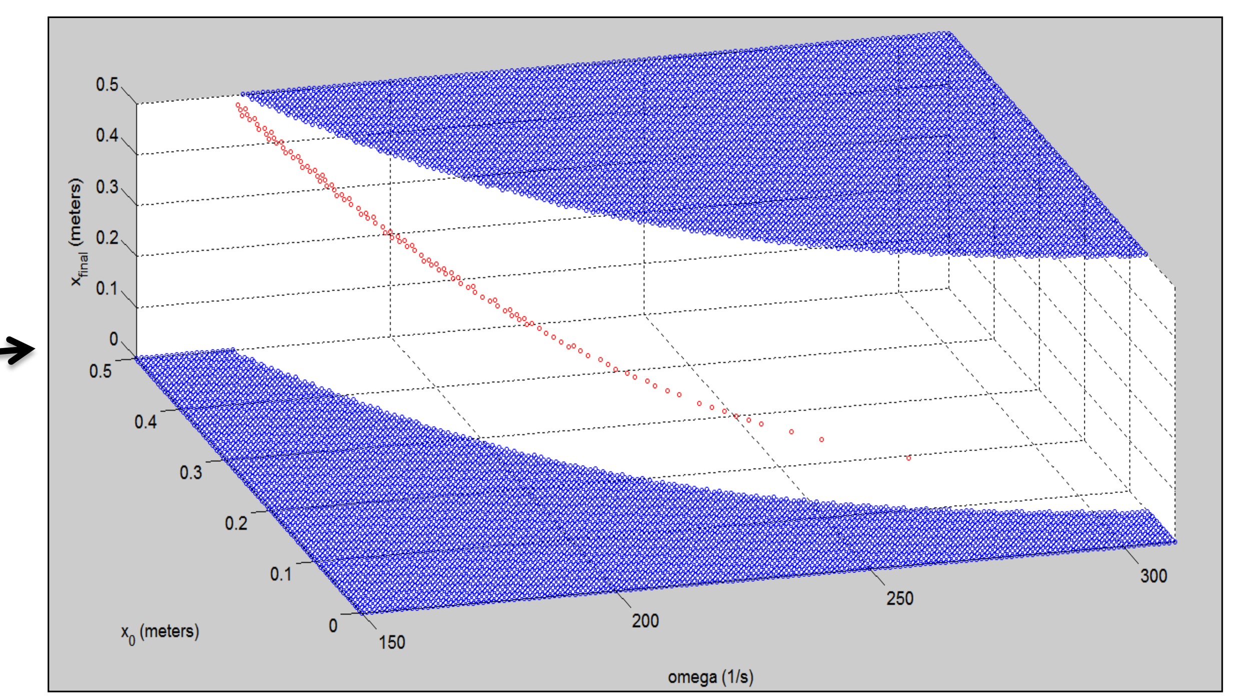
Unstable Solutions from analytical solution and computational solution.

Glossary of Technical Terms

Buoyancy: Force due to volume of displaced water.
Added Mass: Larger observed inertia of bubble due to displacement of surrounding water.
Quasi-static: Temperature does not change enough to consider thermodynamic effects.
Cavitation: The fluid is under enough pressure it does not spontaneously separate.
Turbulent flow: The drag coefficient of the bubble is constant.



Results from analytical computational solutions for one initial depth and three different angular frequencies.



Bifurcation diagram of numerical solution over all physical values of initial position and angular frequency of vibration.

References

1. Sorokin, V. S., Blekhman, I. I., Vasilkov, V. B.: *Motion of a gas bubble in fluid under vibration*. Nonlinear Dyn (2012) 67:147–158.
2. Blekhmann, I.I.: *Vibrational Mechanics.* World Scientific, Singapore (2000)
3. 2013, Avanced Cooling Technologies, Inc., *Oscillating Liquid Cooling*, http://www.1-act.com/advanced-technologies/pumped-liquid-cooling/oscillating-liquid-cooling/ (April 24, 2013)