Energy Flow in Electrical Grids

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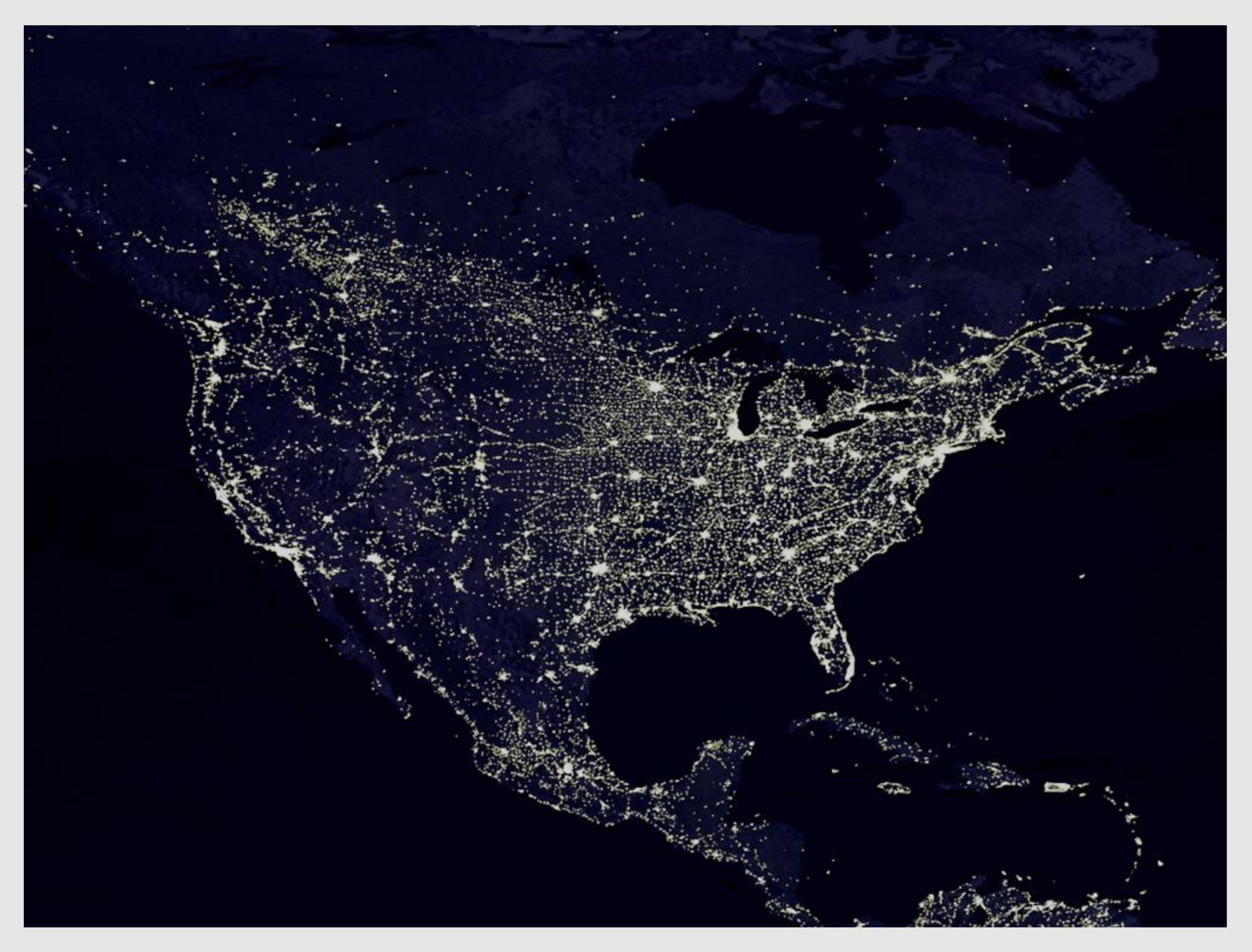
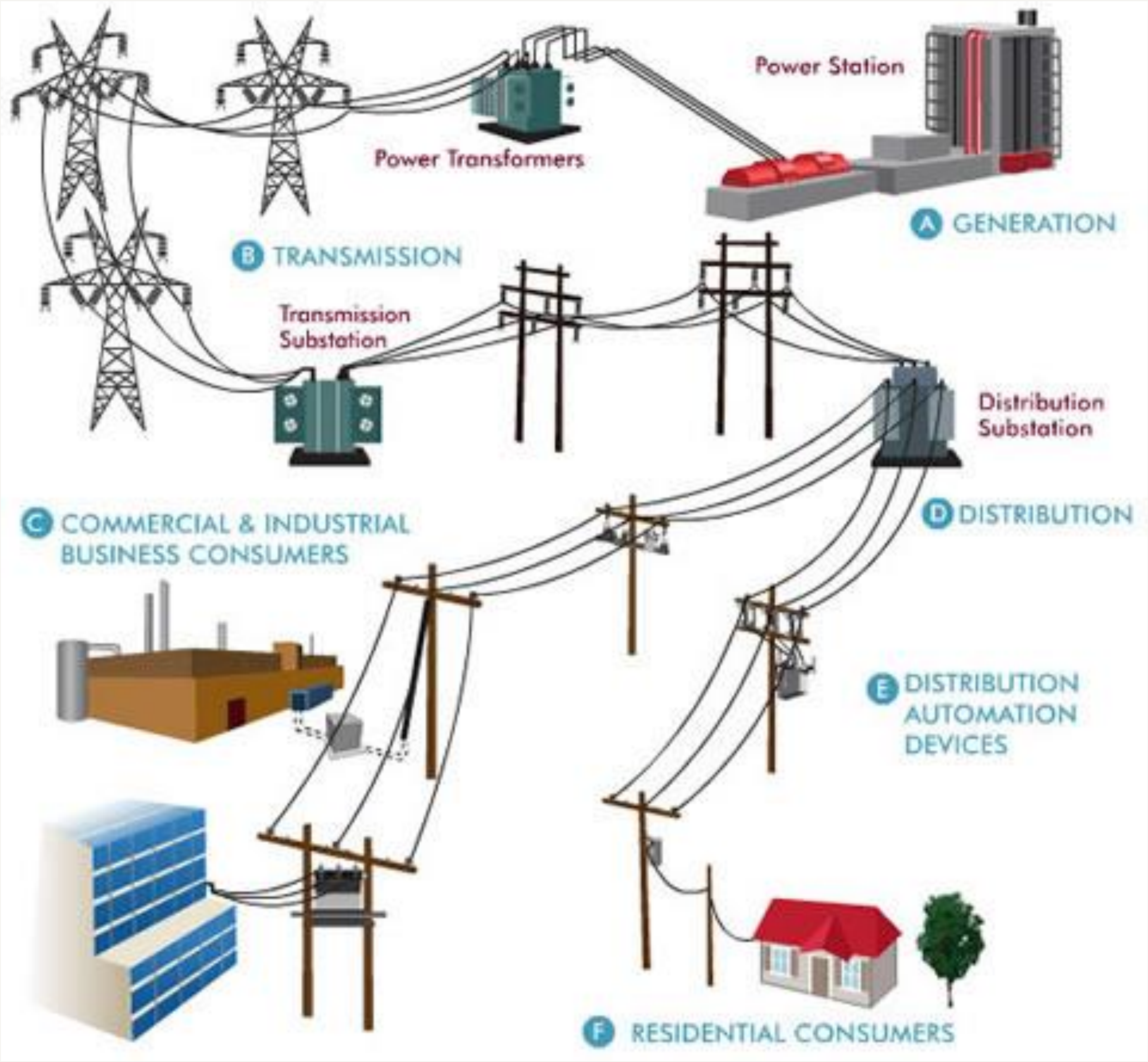


Overview



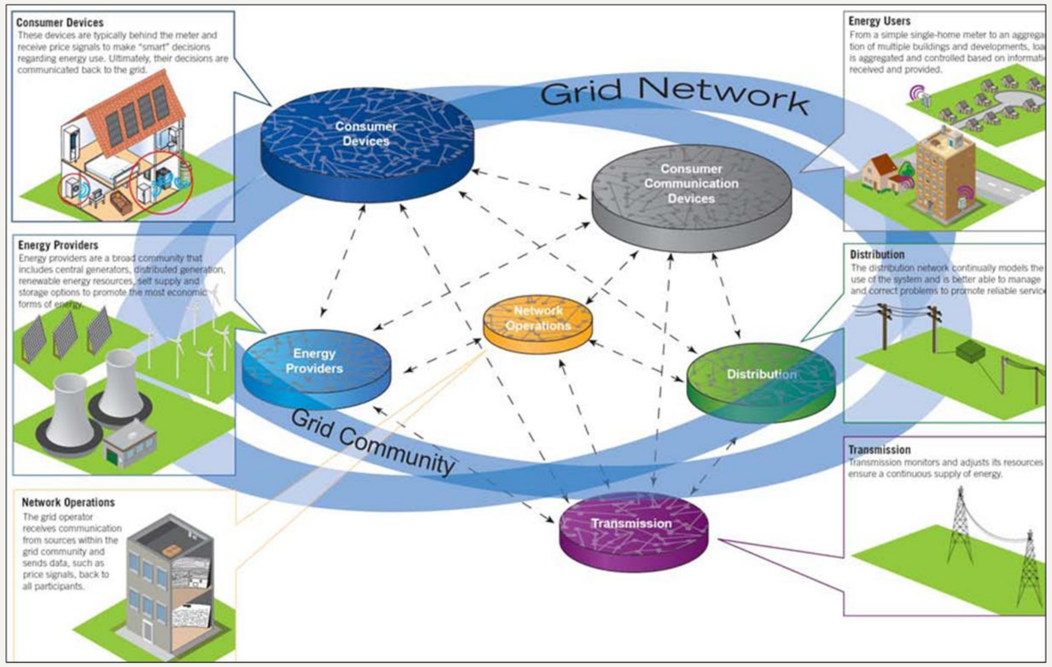
http://www.altenergymag.com/articles/09.04.01/smartgrid/grid.jpg

Motivation

- Along a feed line voltage fluctuates
    - Desire: Reduce voltage drop at end of line
- Determine maximum length supporting consumption
- Goal: Create low-parametric model of power and voltage distribution along an electrical line using [1] to understand energy flow in grids
- Adjust model to include consumption variations in loads

[1] DistFlow ODE: Modeling, Analyzing and Controlling Long Distribution Feeder by Danhua Wang, Konstantin Turitsyn, and Michael Chertkov

Applications



Smart Grids

PV systems

Transporting Energy

Superbowl Power Outage

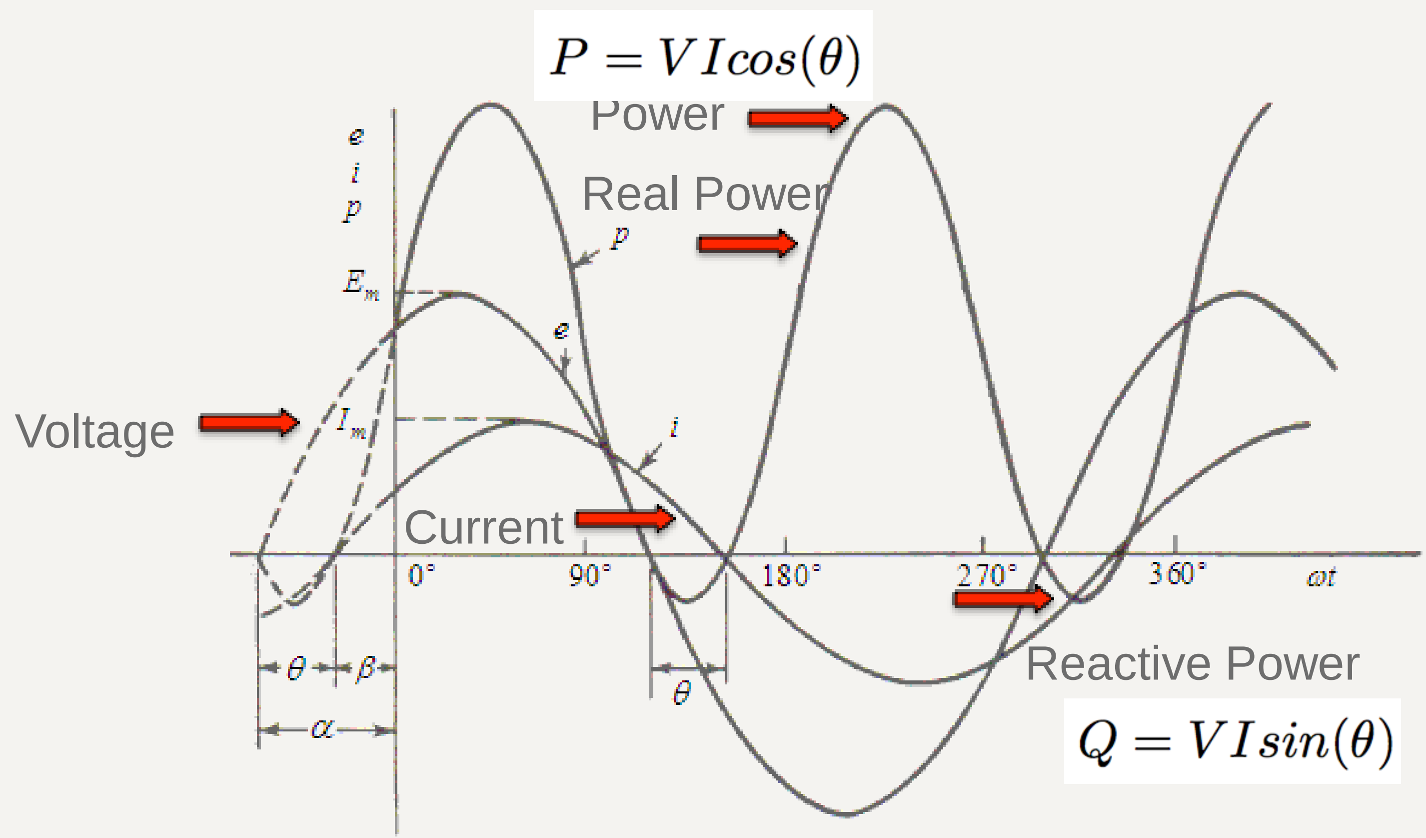
Background

- Alternating Current

$$v(t) = \sqrt{2}V\,sin(\omega t + \alpha) \qquad i(t) = \sqrt{2}I\,sin(\omega t + \beta)$$

$$p(t) = VIcos(\alpha - \beta) - VIcos(2\omega t + \alpha + \beta)$$

v = Voltage
i = Current
p = Power



Background

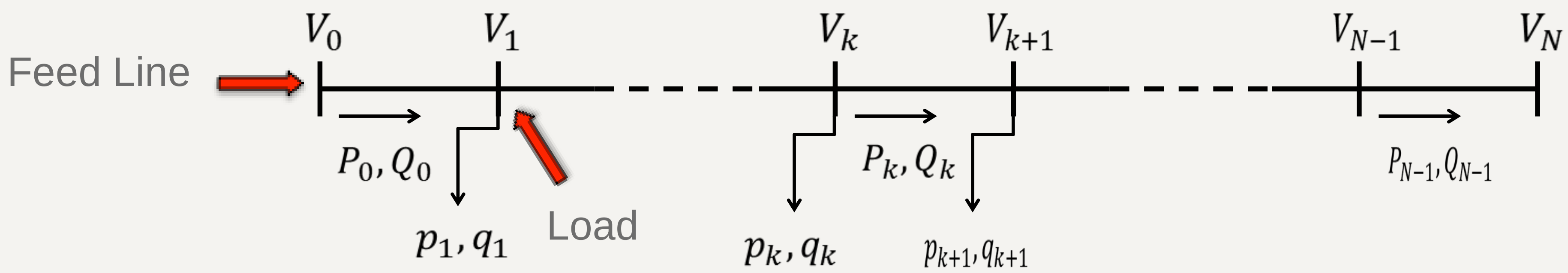
- Other Basic Equations

$$S = VI = P + jQ$$

$$z = r + jx$$

S = Apparent Power
z = impedance
x = inductance
r = resistance

Set-up



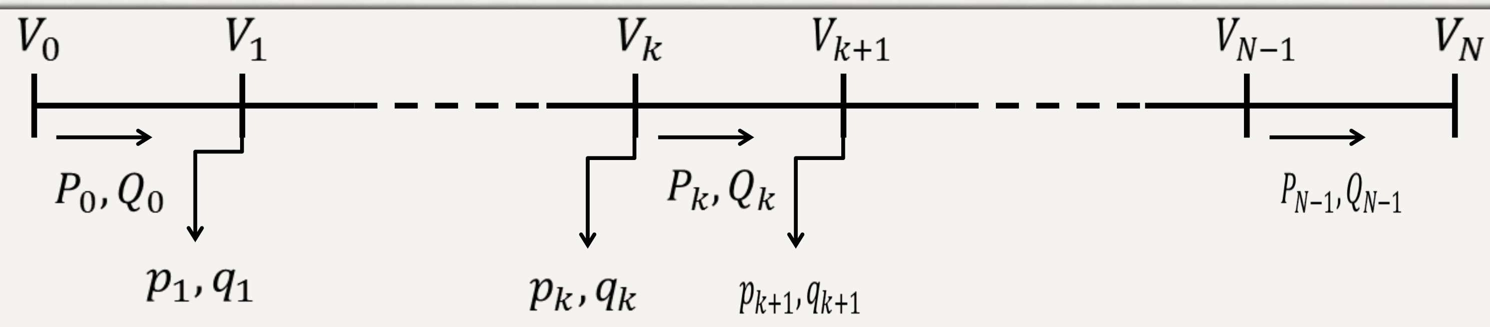
$$S_1 = S_0 - S_l - S_L$$

$$V_1 = V_0 - z_1 I_0$$

Assumptions: One directional flow, uniform consumption at loads, static

Developing the Model

Problem Formulation



Discrete form

$$
\begin{aligned}
P_{k+1} - P_k &= p_k - r_k \frac{P_k^2 + Q_k^2}{v_k^2} \\
Q_{k+1} - Q_k &= q_k - x_k \frac{P_k^2 + Q_k^2}{v_k^2} \\
v_{k+1}^2 - v_k^2 &= -2(r_k P_k + x_k Q_k) - (r_k^2 + x_k^2) \frac{P_k^2 + Q_k^2}{v_k^2}
\end{aligned}
$$

where

$k = 0, ..., N$ enumerates buses of the feeder

P_k,Q_k real and reactive power flowing from bus k to bus $k+1$

p_k, q_k overall consumption of real and reactive power at bus k

r_k, x_k line resistance and reactance connecting bus k to bus $k+1$

with Boundary Conditions

$$v_0 = 1, P_N = Q_N = 0$$

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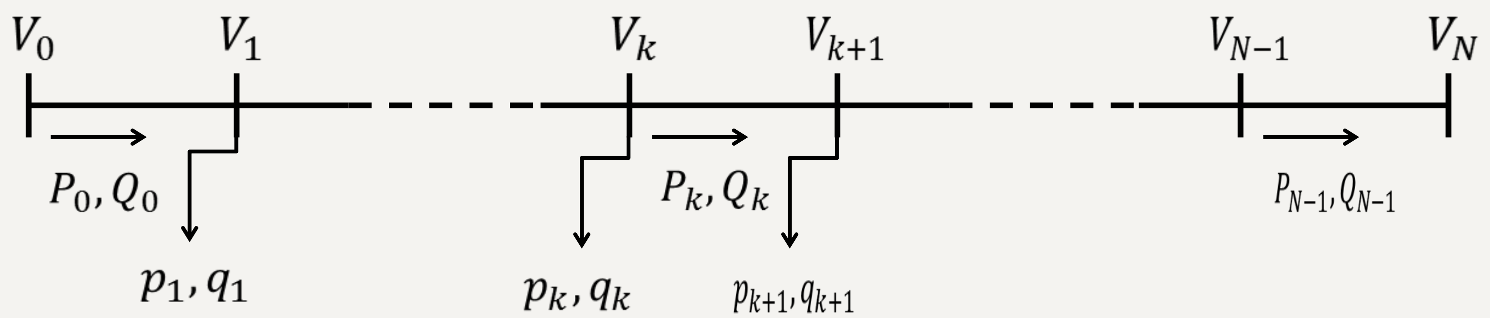
Continuous and Homogenous Form

Transform the discrete finite element to a continuous form:

- assume large number of consumers $N \gg 1$
- continuous form with limit $N \longrightarrow \infty$
- $\frac{r_k}{x_k}$ is set constant so $r_k = r\frac{l_k}{L}$and $x_k = x\frac{l_k}{L}$
- r and x are constant values
- L total length of the feeder line and l_k length of line from bus k to bus $k+1$
- $F_k = F(z) + \tilde{F}(L_k)/N$
  - $F(z)$ which is the change from node k to $k+1$
  - $\tilde{F}(L_k)/N$ which is the averaging term
- $z = L_k = \sum_{i=0}^{k-1} l_k$
- $F_{k+1} - F_k \approx F'(z)l_k/L$

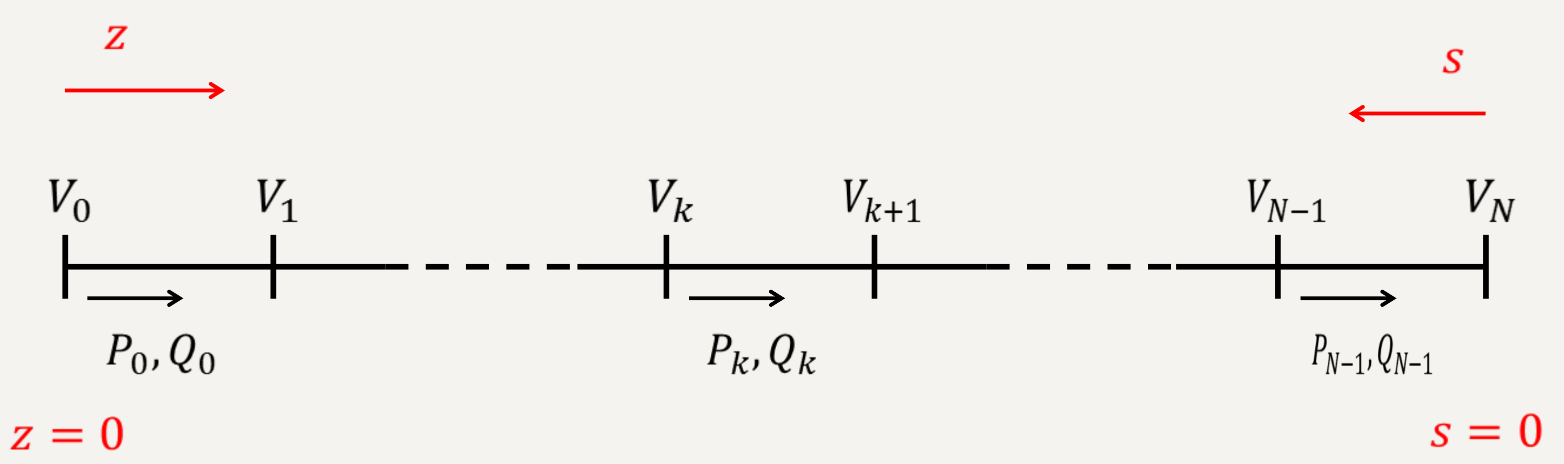


Boundary Value Problem

$$\frac{d}{dz}\begin{pmatrix} P \\ Q \\ v \end{pmatrix} = \begin{pmatrix} p - r\frac{P^2+Q^2}{v^2} \\ q - x\frac{P^2+Q^2}{v^2} \\ -\frac{rP+xQ}{v} \end{pmatrix}$$

with Boundary Conditions

$$v_0 = 1, P(L) = Q(L) = 0$$



Re-scaled Form

- Assuming p =constant
- New Variable $s = \dfrac{\sqrt{|p|r}}{v(L)}(L - z)$

Dimensionless Variables for P, Q and v

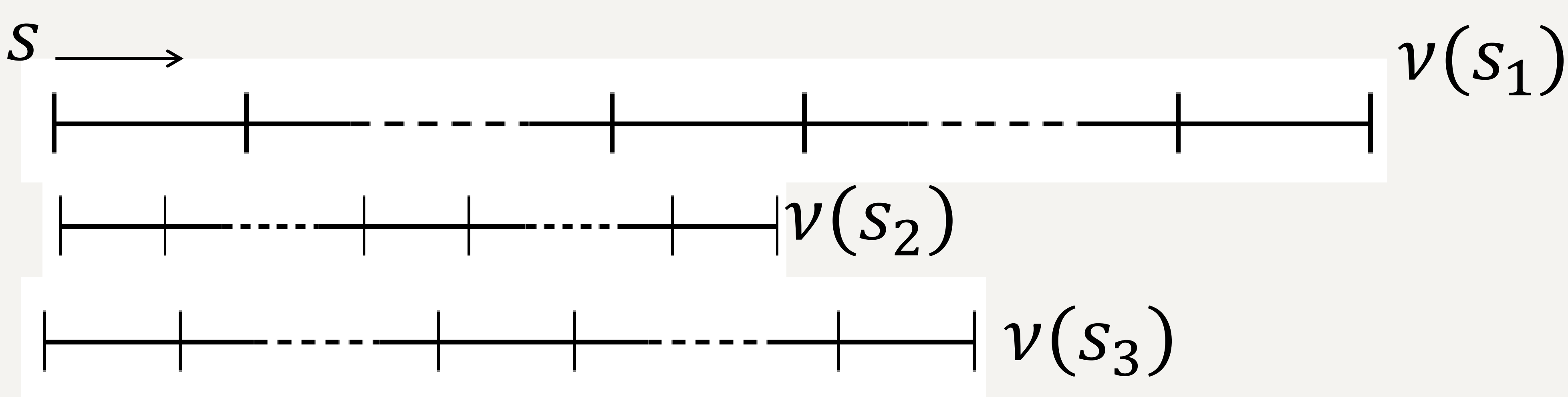
$$\varrho(s) = \sqrt{\frac{r}{|p|}}\frac{P(z)}{v(L)}, \tau(s) = \sqrt{\frac{r}{|p|}}\frac{Q(z)}{v(L)}, \upsilon(s) = \frac{v(z)}{v(L)}$$

Initial Value Problem

$$\frac{d}{ds}\begin{pmatrix} \varrho \\ \tau \\ \upsilon \end{pmatrix} = \begin{pmatrix} sign(p) - \frac{\varrho^2+\tau^2}{\upsilon^2} \\ A - B\frac{\varrho^2+\tau^2}{\upsilon^2} \\ -\frac{\varrho+B\tau}{\upsilon} \end{pmatrix}$$

with Initial Conditions

$$\upsilon(0) = 1, \varrho(0) = \tau(0) = 0$$



Evaluating for End Points

Solving IVB for some value of s_*
$s : 0 \to s_*$ we obtain $\varrho(s_*)$, $\tau(s_*)$ and $\upsilon(s_*)$
Then we can compute the value of L and the end values

$$L = \frac{s_*}{\upsilon(s_*)\sqrt{|p|r}}$$

$$\upsilon(L) = \frac{1}{\upsilon(s_*)}$$

$$P(0) = \frac{\varrho(s_*)\sqrt{|p|/r}}{\upsilon(s_*)}$$

$$Q(0) = \frac{\tau(s_*)\sqrt{|p|/r}}{\upsilon(s_*)}$$

Reproduction of Results

Initial and Boundary Value Problems

- IVP

$$-\frac{d}{ds}\begin{pmatrix}\rho\\ \tau\\ \upsilon\end{pmatrix}=\begin{pmatrix}\operatorname{sign}(p)-\frac{\rho^2+\tau^2}{\upsilon^2}\\ A-B\frac{\rho^2+\tau^2}{\upsilon^2}\\ -\frac{\rho+B\tau}{\upsilon}\end{pmatrix}$$

BVP

$$\frac{d}{dz}\begin{pmatrix}P\\ Q\\ v\end{pmatrix}=\begin{pmatrix}p-r\frac{P^2+Q^2}{v^2}\\ q-x\frac{P^2+Q^2}{v^2}\\ -\frac{rP+xQ}{v}\end{pmatrix}$$

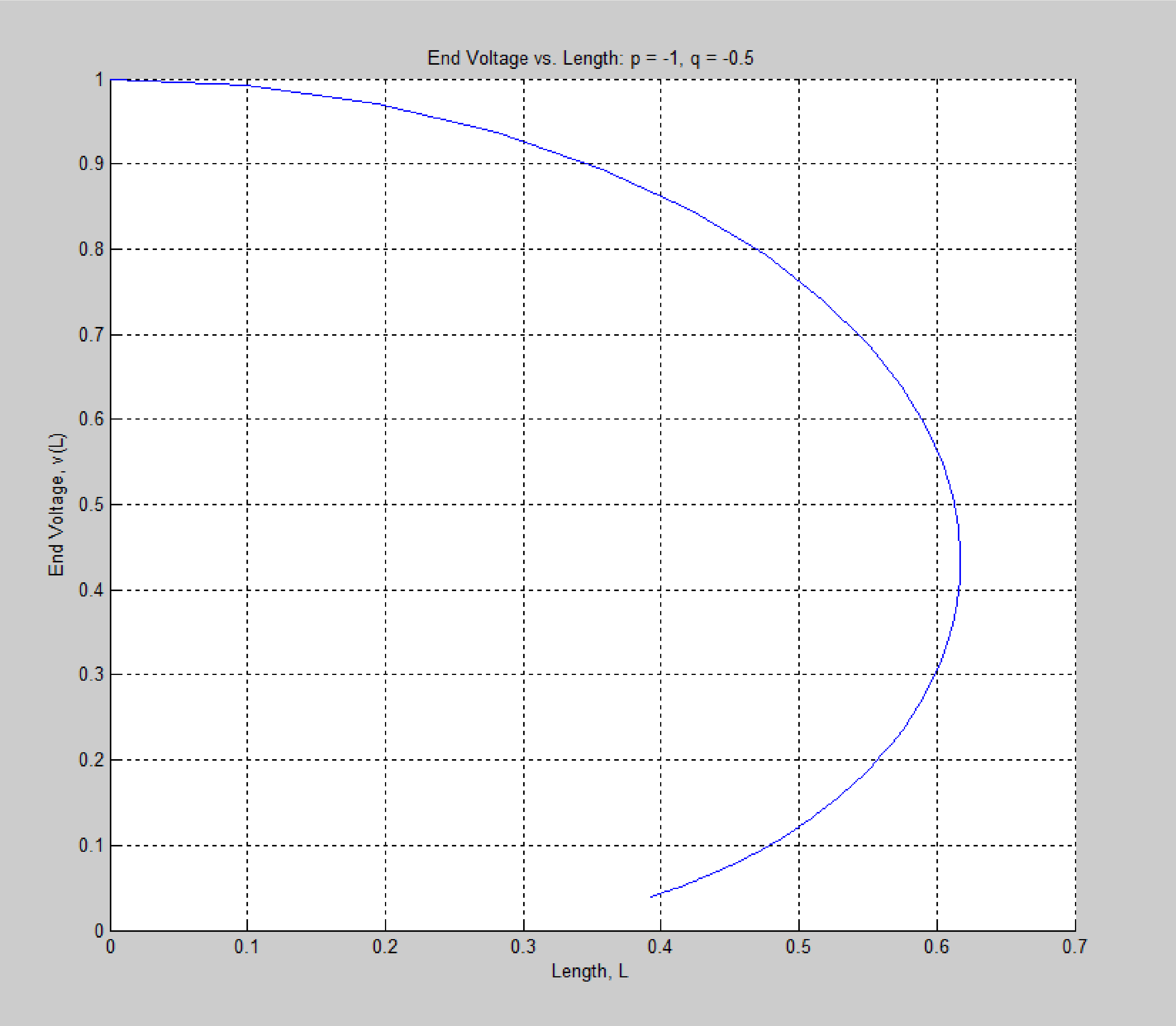
Graphs generated from each problem:

- End Voltage vs. Length
- Power Utilization vs. Length

Voltage vs. Position

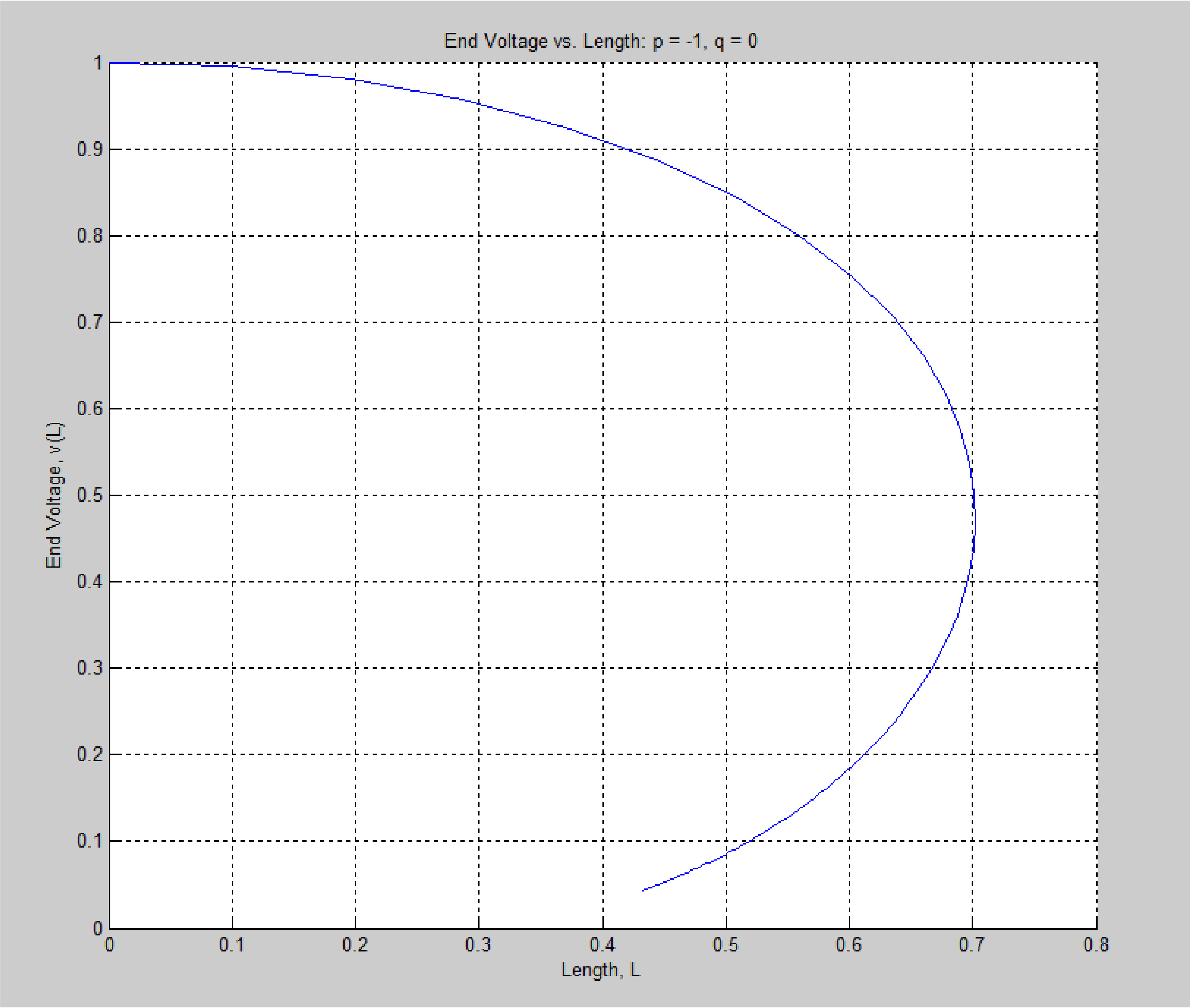
Power vs. Position

End Voltage vs. Length
p = -1, q = -0.5



End Voltage vs. Length

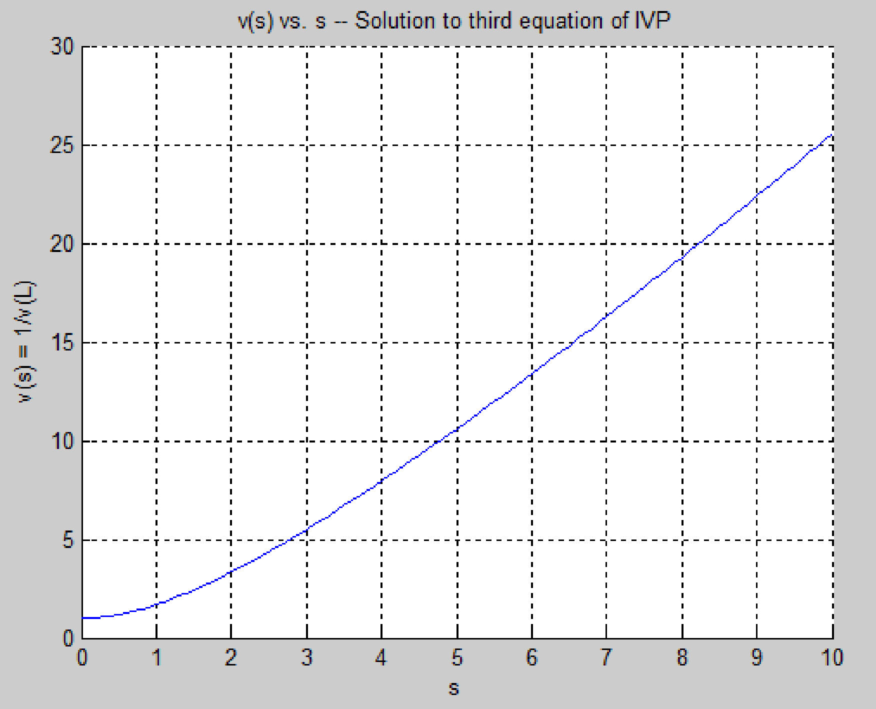
p = -1, q = 0

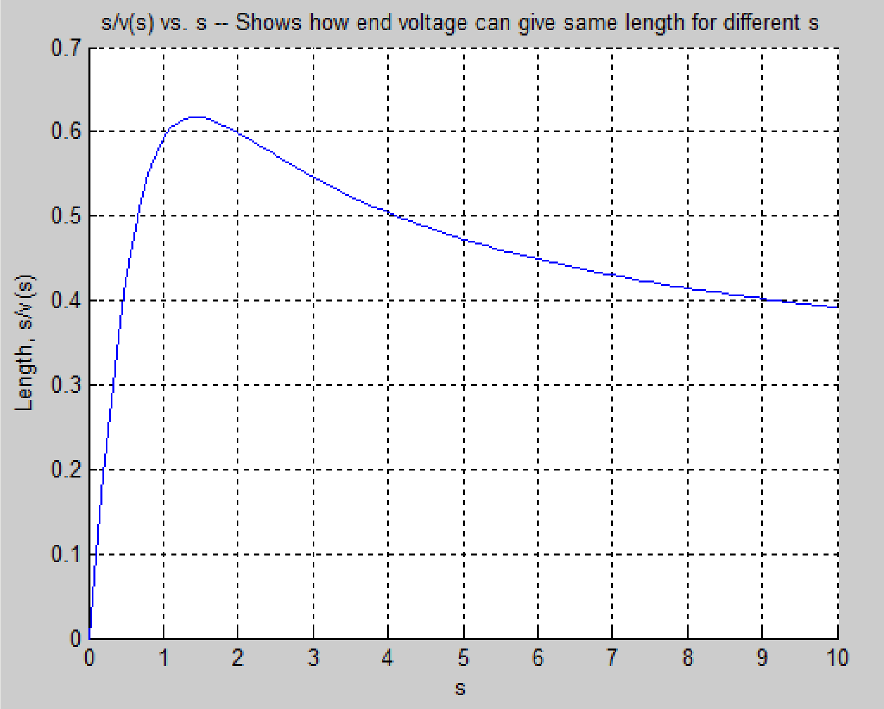


Two End Voltages for One Length

$$v(L) = \frac{1}{\upsilon(s_*)}$$

$$L = \frac{s_*}{\upsilon(s_*)\sqrt{|p|r}}$$





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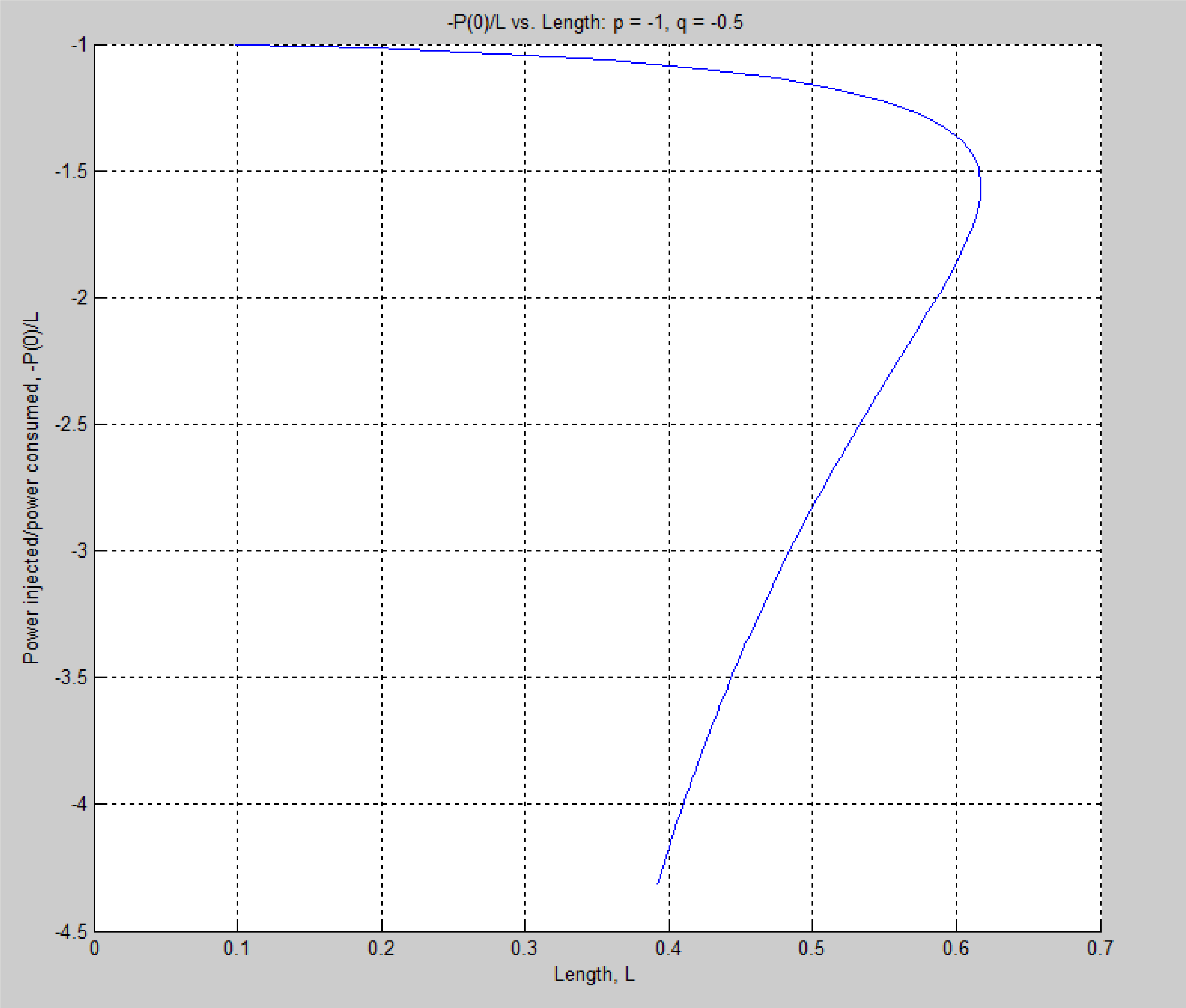
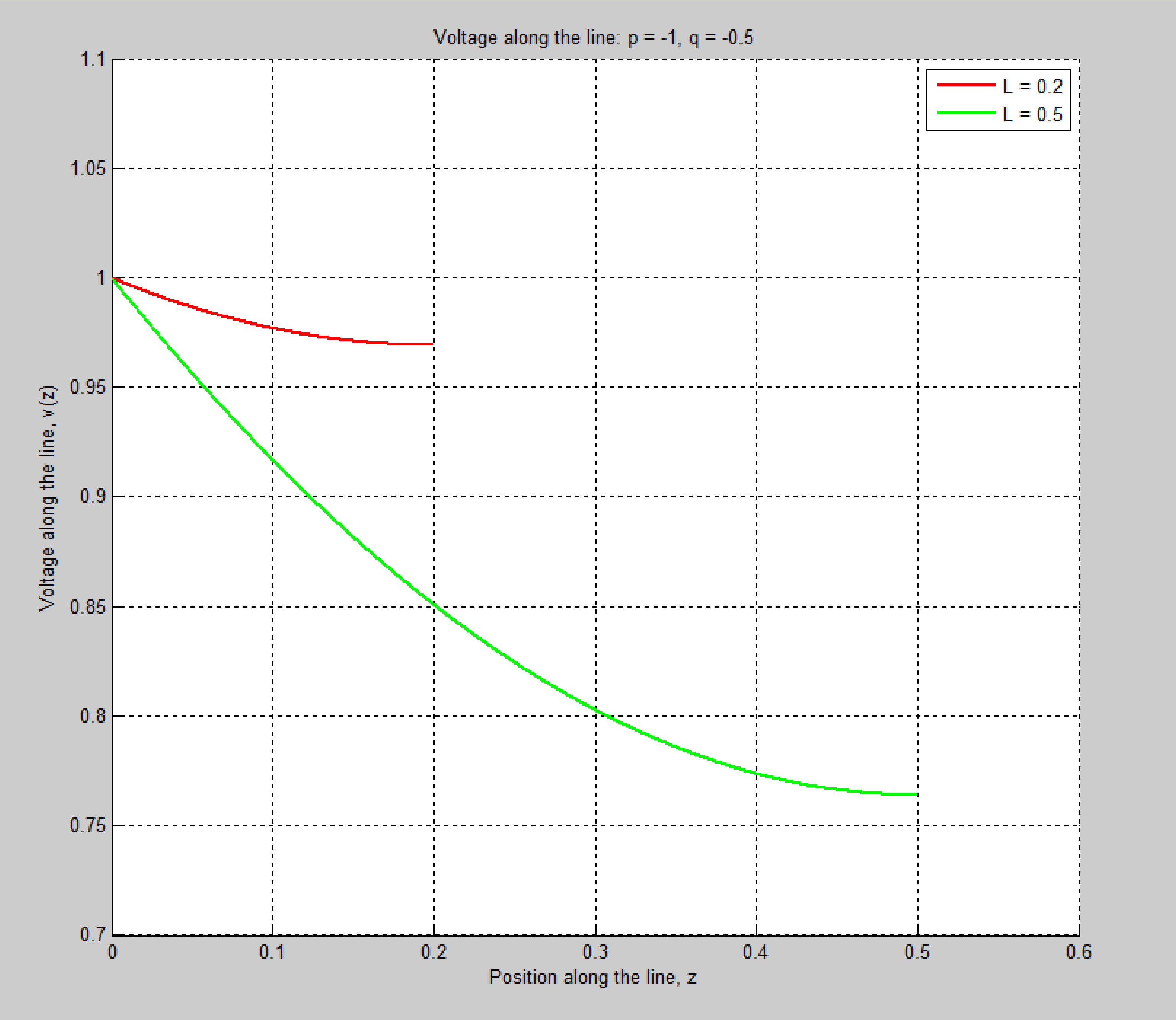
Power Utilization vs. Length

$$\text{Power utilization} = \frac{\text{Initial Injected Power}}{\text{Power Consumed}} = \frac{P(0)}{p*L}$$

Power Utilization vs. Length
p = -1, q = -.5



Voltage Along the Line



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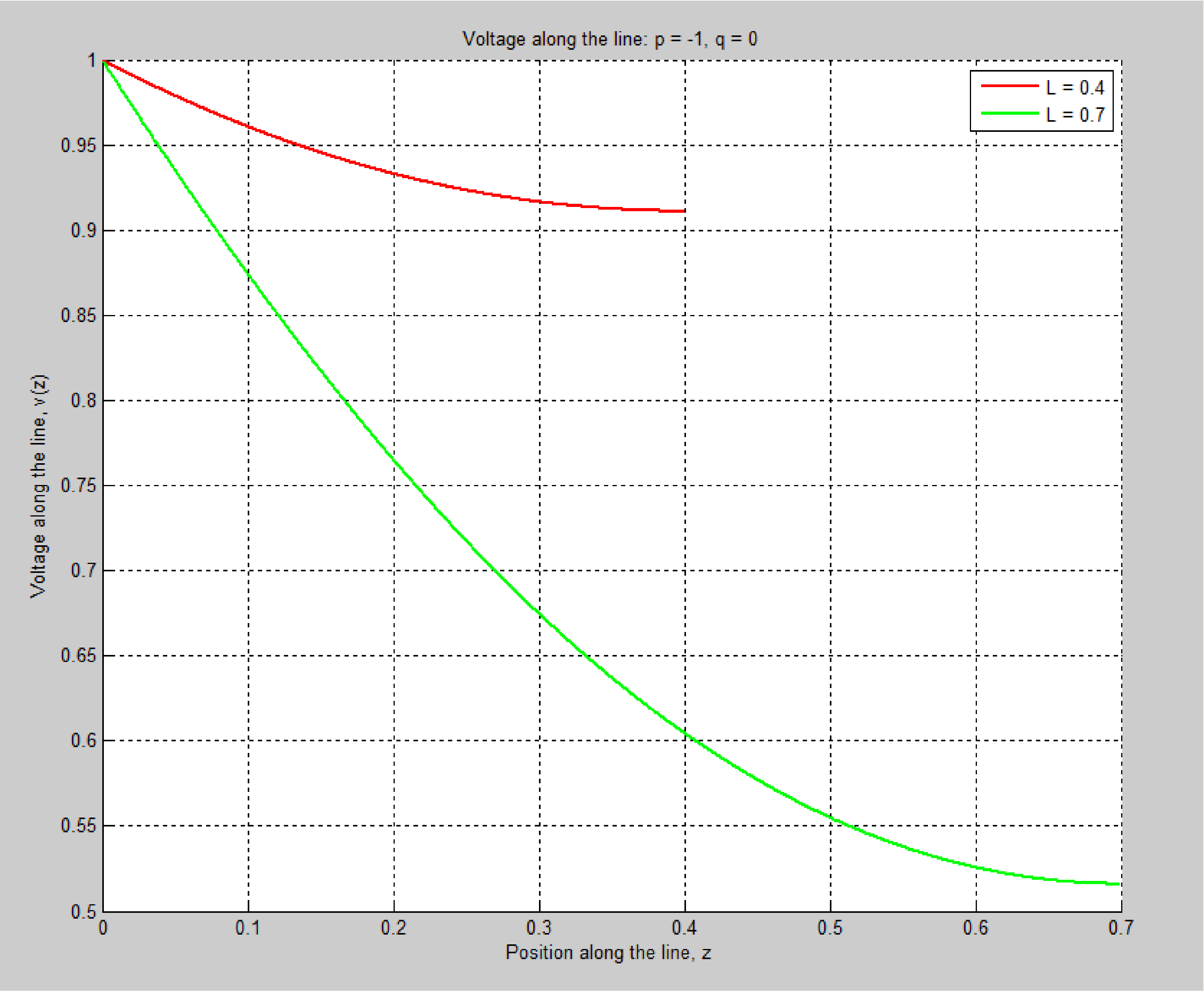
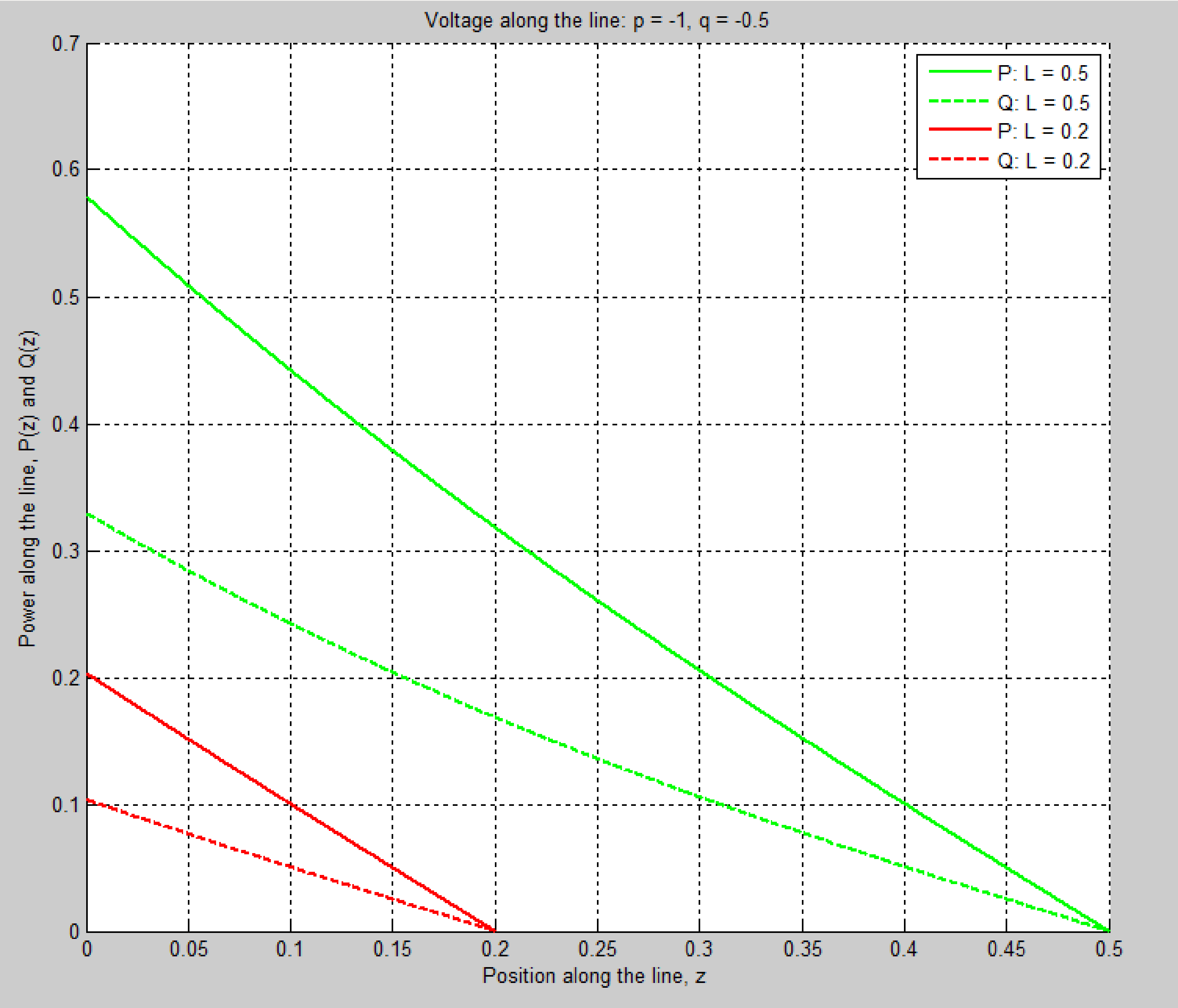
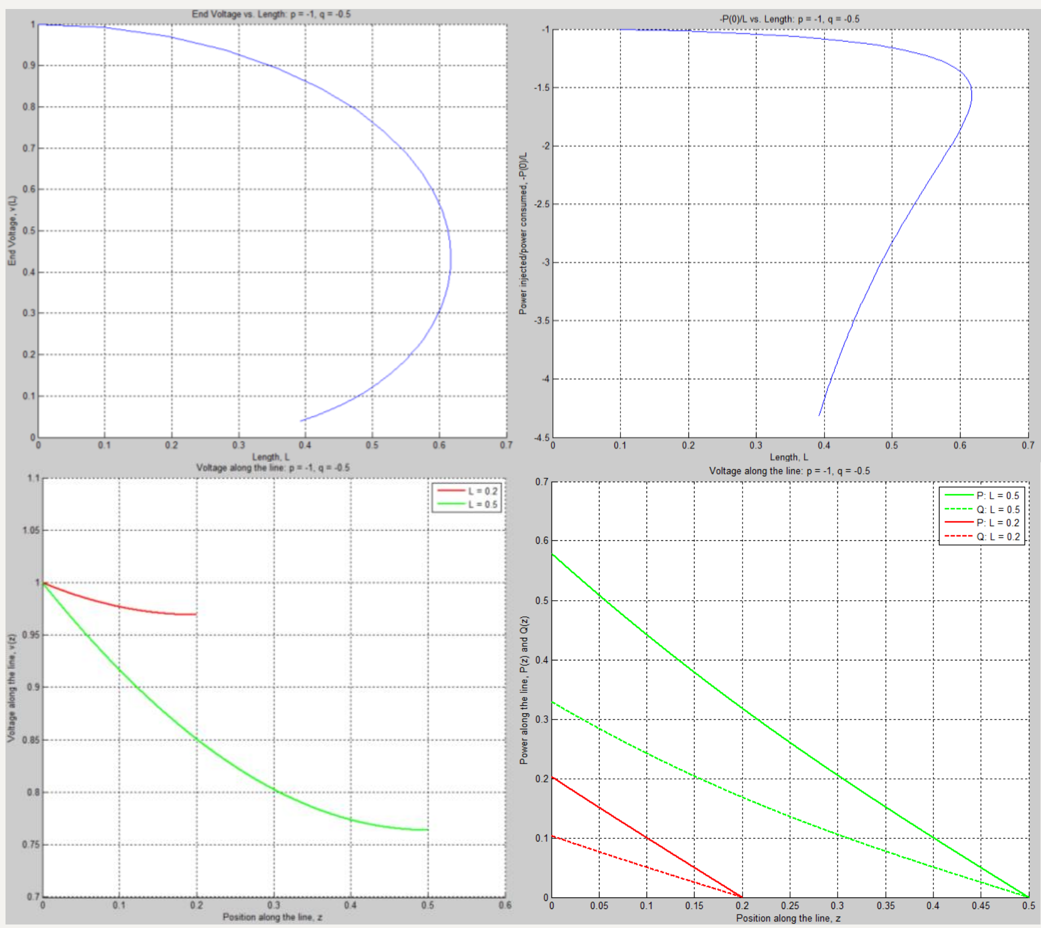
Voltage Along the Line

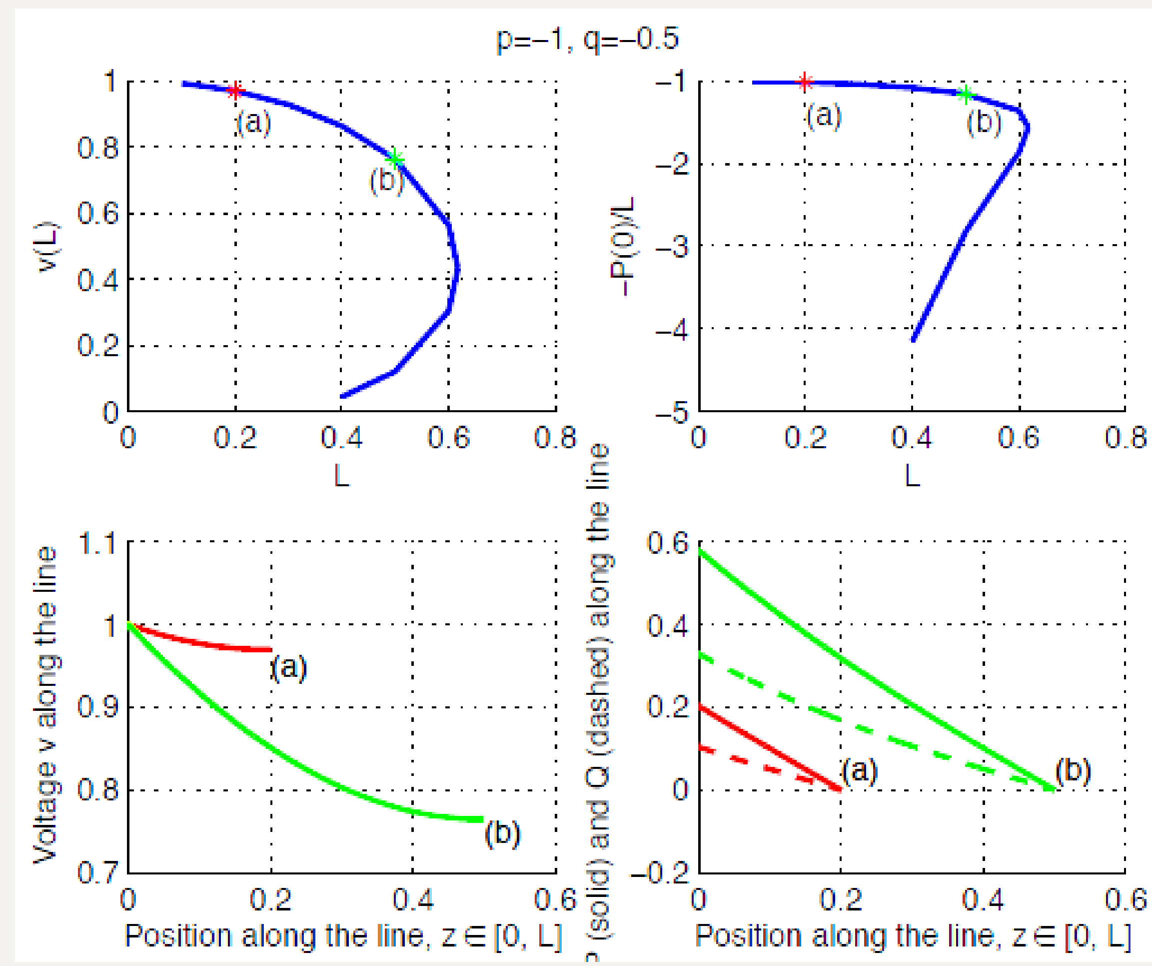


Power Along the Line



Our graphs vs. Article

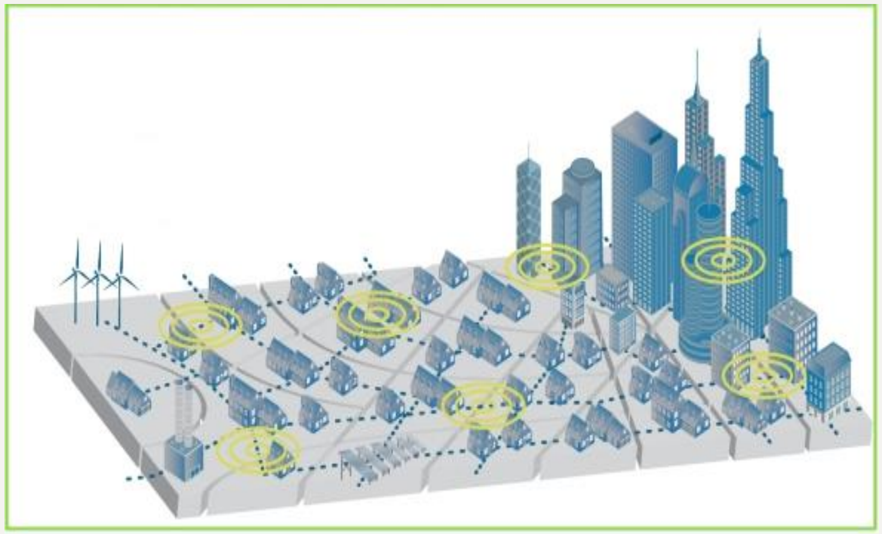




Stochastic Addition

Non-uniform Consumption

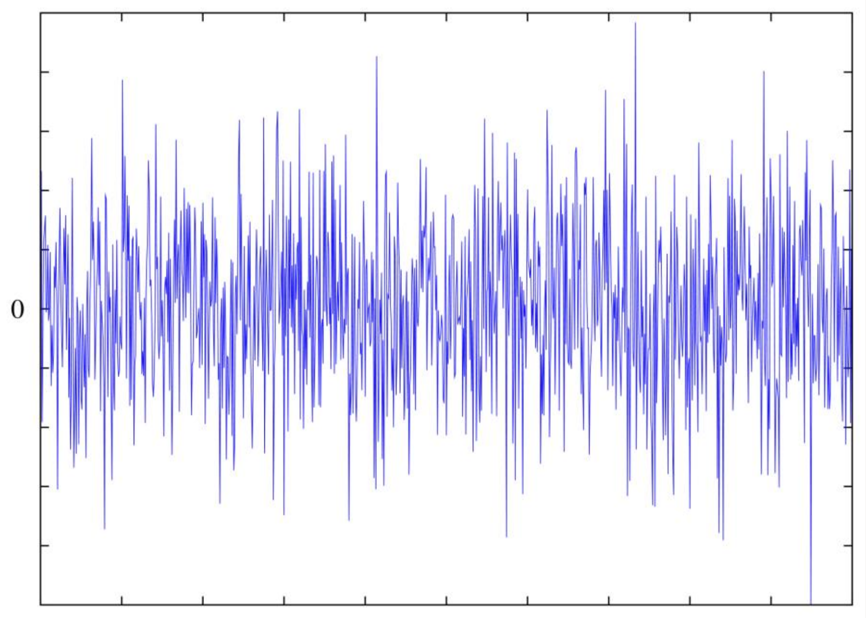
- Major assumption in the article:
  - Uniform consumption of loads p
- In reality
  - Slight variations across a line



Adding Stochasticity

- Let $p(l) = p_0 + W(l)$, where

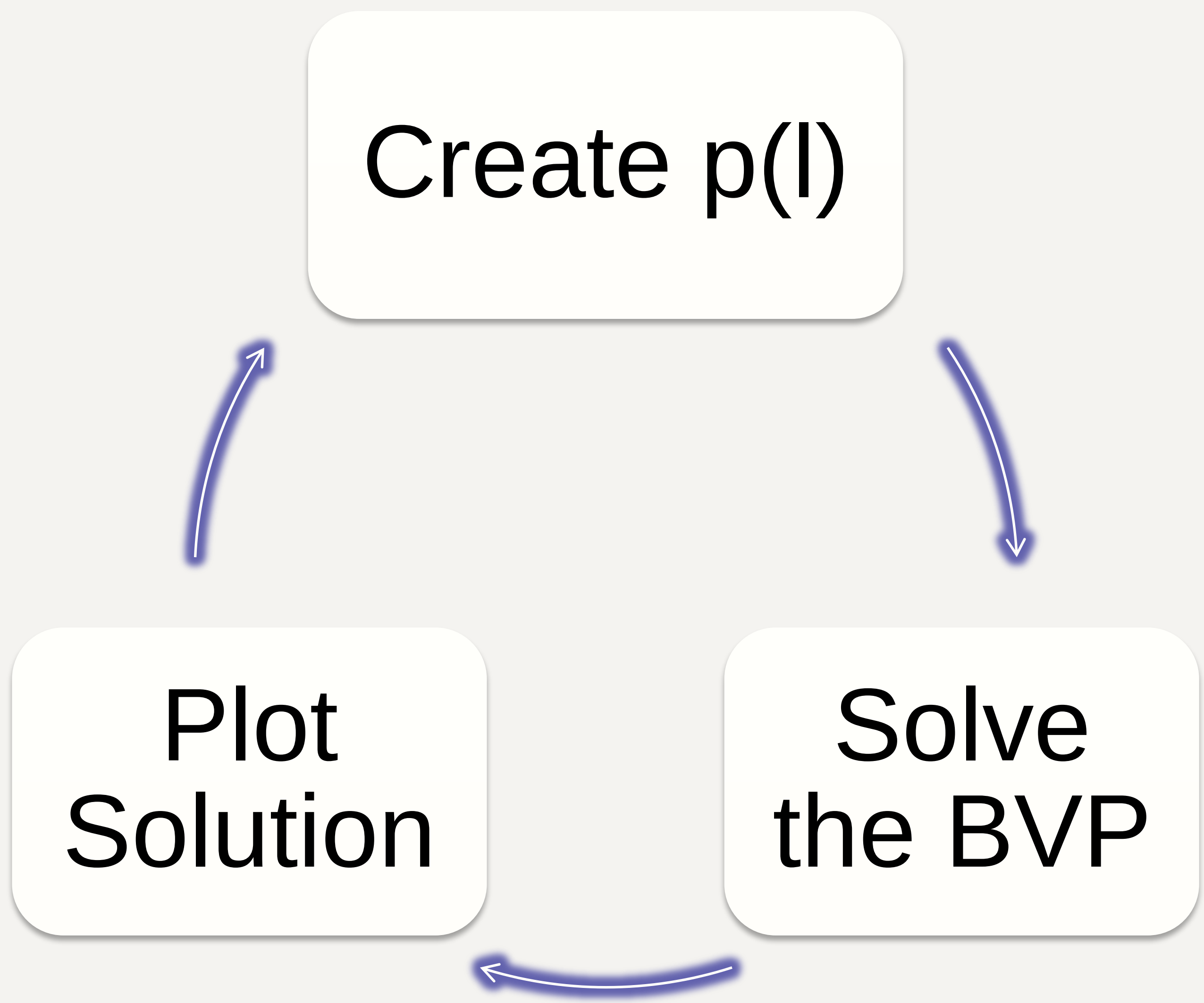
  $W(l)$ is a Wiener Process.



  - Substitute for p in DistFlow ODEs
  - Solve for boundary value problem

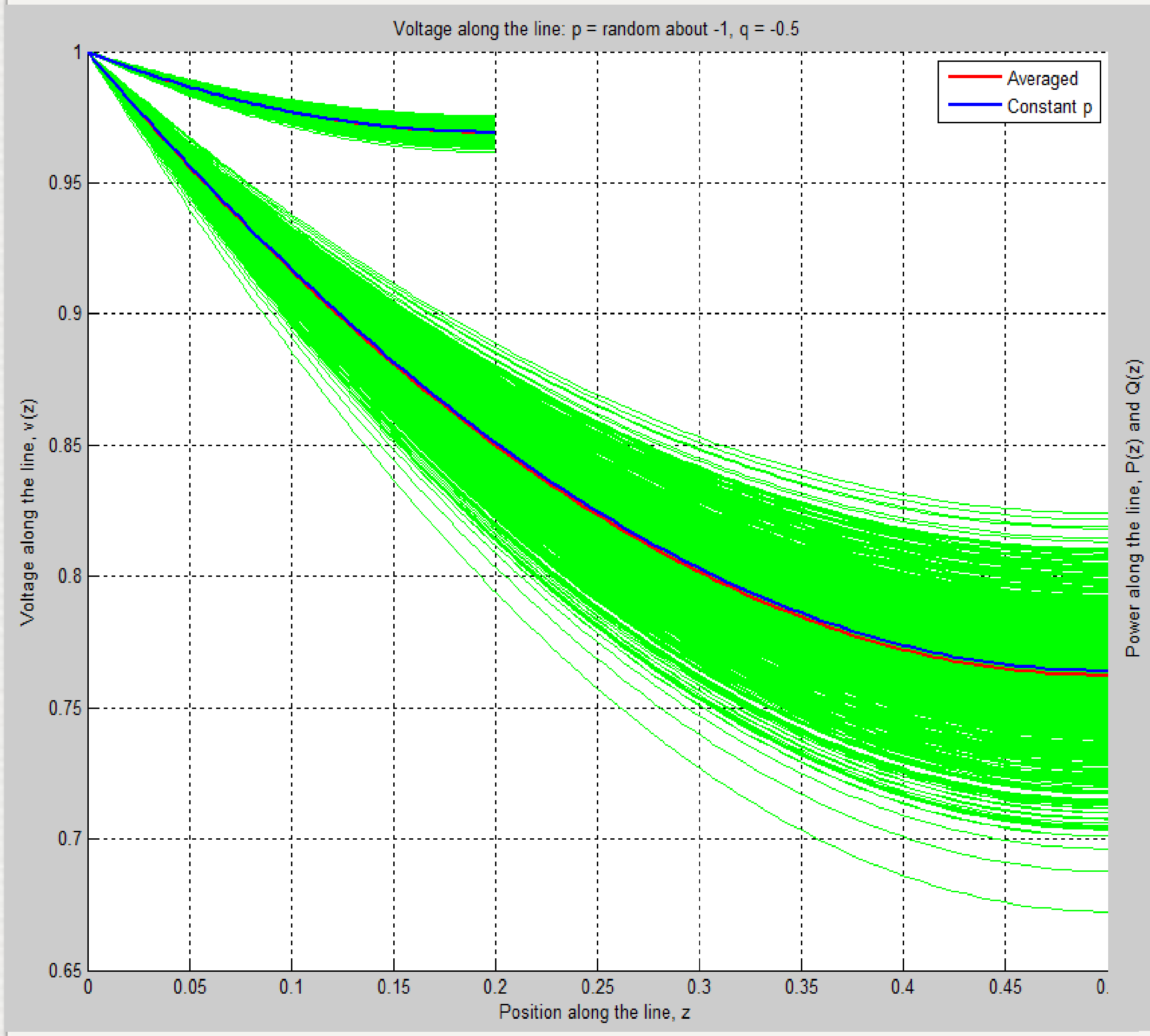
Methodology

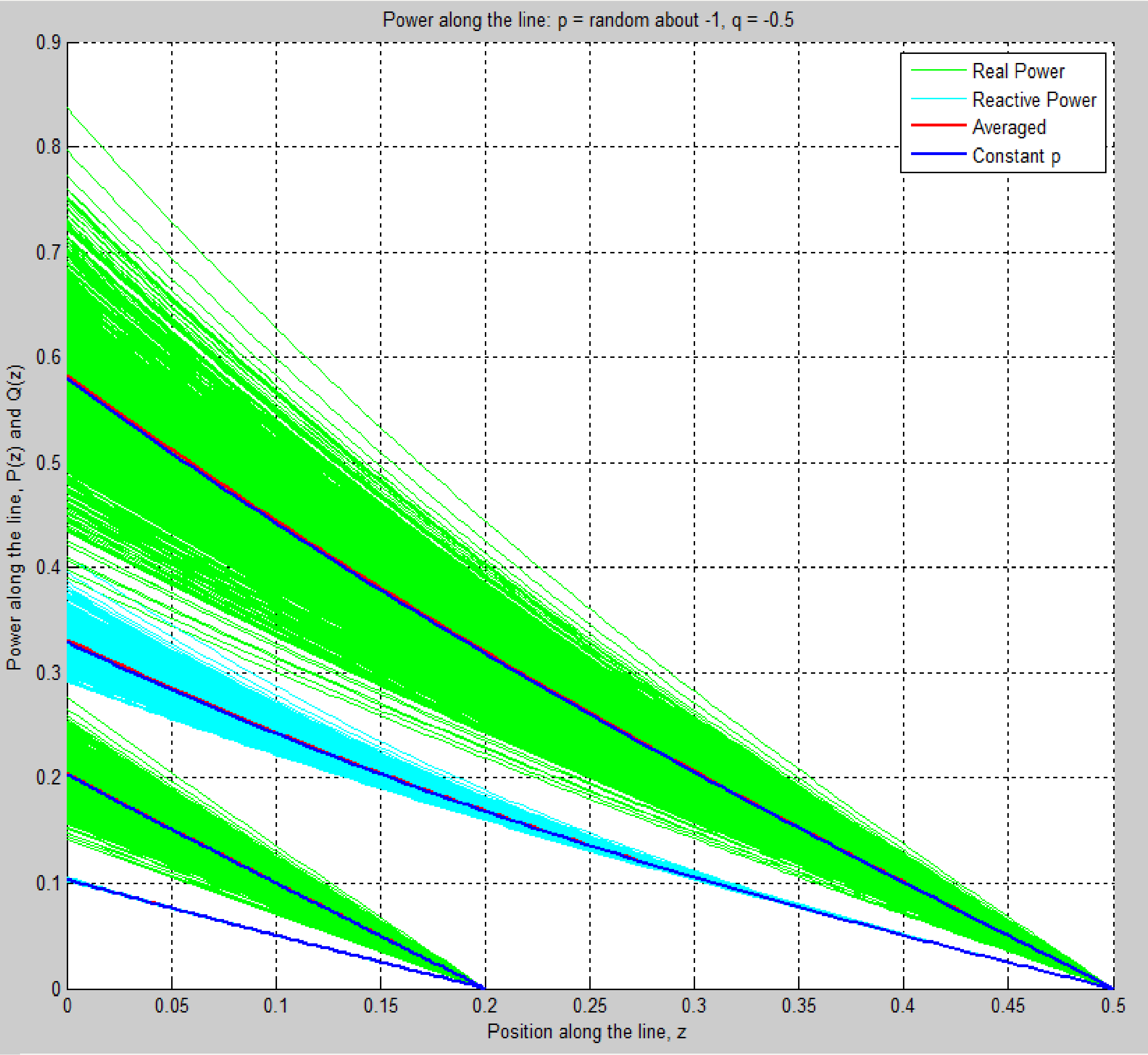
- Monte Carlo Method



Results

Results


Voltage along the line: p = random about -1, q = -0.5
Averaged
Constant p
Voltage along the line, v(z)
Position along the line, z


Power along the line: p = random about -1, q = -0.5
Real Power
Reactive Power
Averaged
Constant p
Power along the line, P(z) and Q(z)
Position along the line, z

Discussion

- Small perturbations of power consumption have relatively little effect on voltage at the end of the line
- Assuming a constant power consumption is statistically valid
- Suggests that power consumption alone does not serve as an explanation to sudden voltage drops

Future Work

Dynamic Model

- Implementation energy variations with respect to time
- Better understanding of 'jumps' in stability
- Introducing producers along the line; effect of renewable energy sources

Branching

- Take into consideration the grid layout of energy distribution systems
- Use discrete form of the model to examine energy flow dynamics of three and four bus systems

References

- The papers used for this presentation were:
- M. Baran and F. Wu, “Optimal sizing of capacitors placed on a radial distribution system,” *Power Delivery, IEEE Transactions on*, vol. 4, no. 1, pp. 735 – 743, jan 1989.
- D Wang, K Turitsyn and M Chertkov, “DistFlow ODE: Modeling, Analyzing and Controlling Long Distribution Feeder”, Proceedings of, the 51st IEEE Conference on Decision and Control (2012) [http://arxiv.org/abs/1209.5776]

Questions?