# Electro-mechanical Properties of Bones

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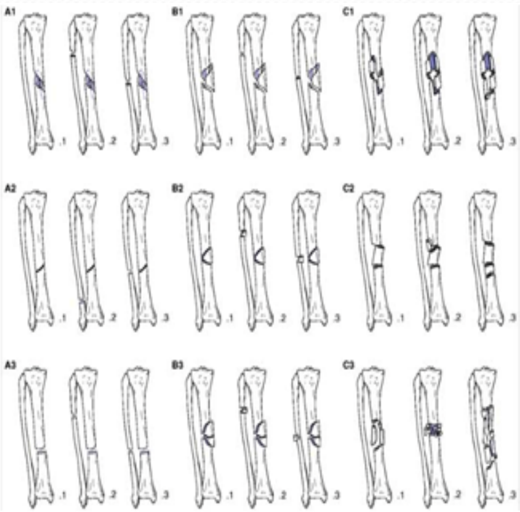
# Introduction

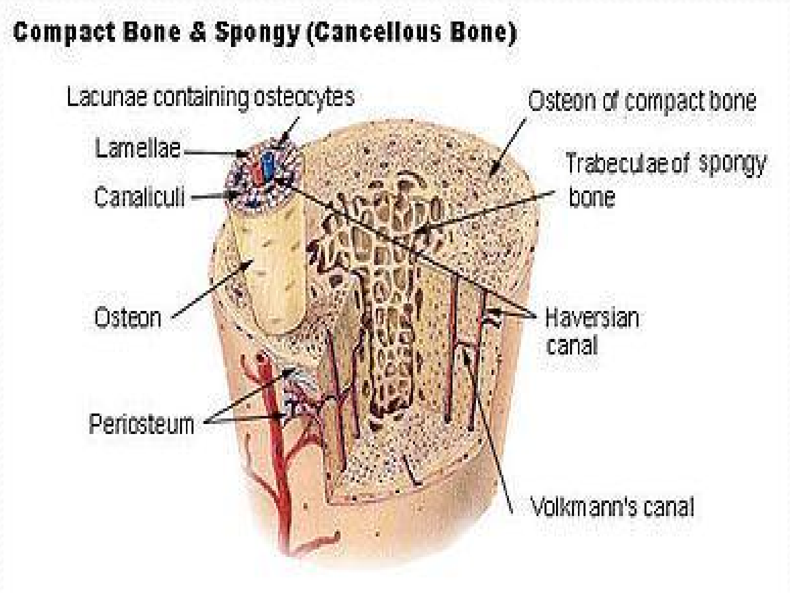
- The role of bones in the human body
  - Support/Protection
  - Production of blood cells
- The study of bone structures
  - Conductive properties of bone
- The properties of piezoelectric materials
- Applicable potential of piezoelectric property of bone
  - Aids in healing process
- Motivation of model



# Properties and Structure of Bones

- Bones contain canals
  - Haversian
  - Volkmann

- Properties of the canals
  - Density/Fill Factor
  - Conductivity



# Modeling the Structure

- Two ways to simplify the structure
  - Figure 1: Conductive ellipsoids running horizontally and vertically, with no interaction between them, in an isotropic medium
  - Figure  2: Conductive ellipsoids in a slightly conductive medium

- Pros/Cons of each model

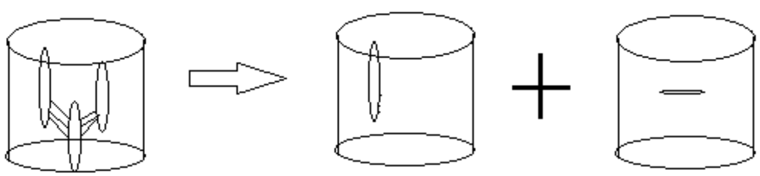


Figure 1

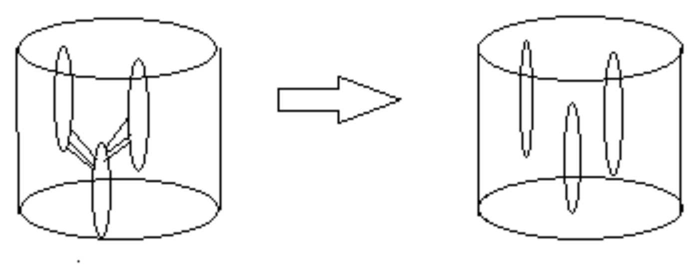
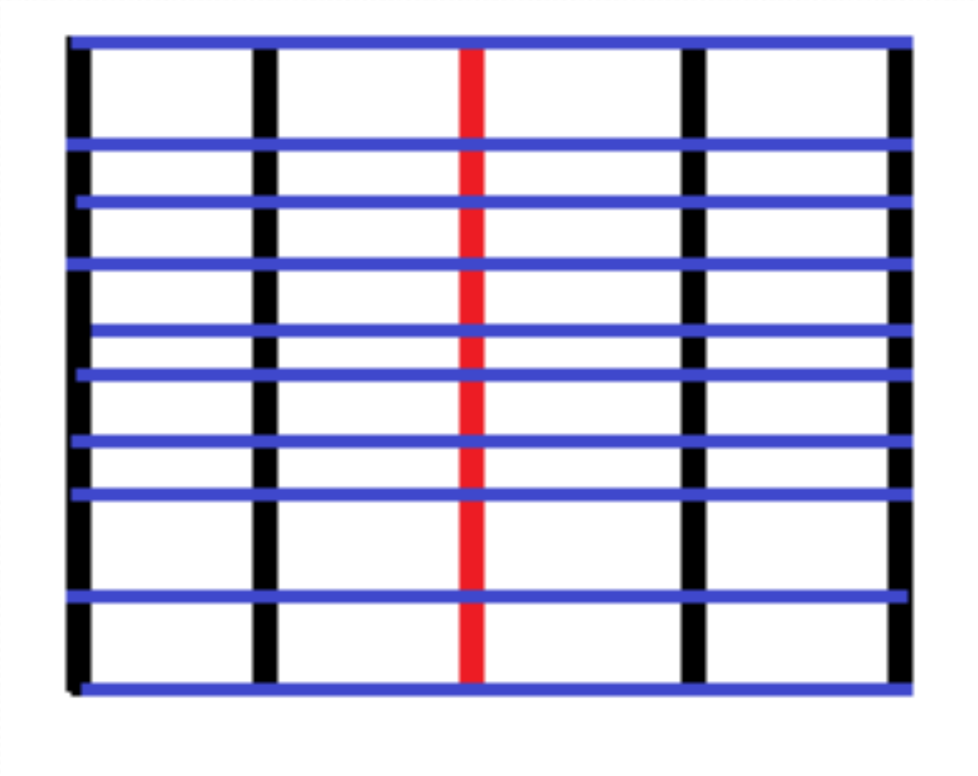


Figure 2

# Numerical Approach



# Numerical Approach (Cont)

Mathematical part

- Using Kirchoff's law a system of linear equations could be made

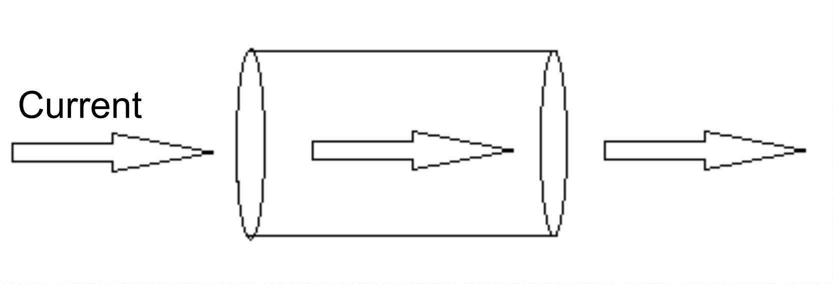
$$\sum_{k=1}^{n} I_k = 0 \text{ and } \sum_{k=1}^{n} V_k = 0$$

- Hence at each node:

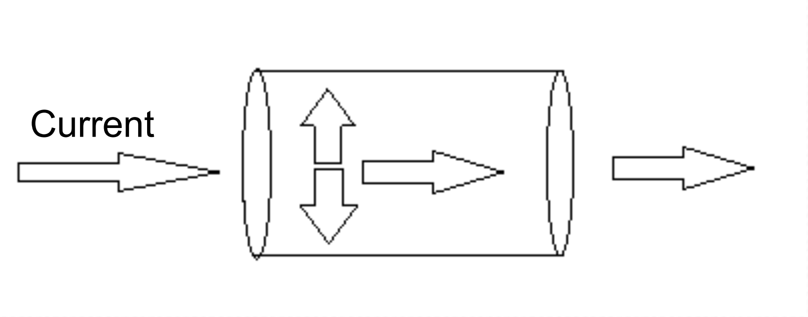
$k_{m,n} = (1/R_1)(k_{m+1,n} - k_{m,n}) + (1/R_2)(k_{m,n-1} - k_{m,n}) + (1/R_3)(k_{m,n} - k_{m-1,n}) + (1/R_4)(k_{m,n} - k_{m,n-1})+...$

# Current Flow Through Bones

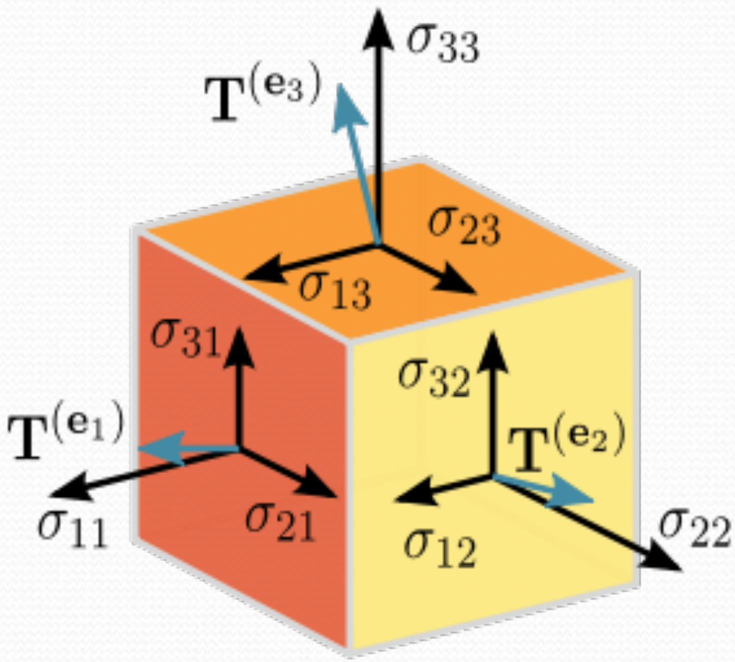
- First consider the simplest case, current through an isotropic metal rod.




- However since bones are anisotropic the flow of current is not so straightforward.



# Tensors




$$\sigma = \left[\mathbf{T}^{(\mathbf{e}_1)}\mathbf{T}^{(\mathbf{e}_2)}\mathbf{T}^{(\mathbf{e}_3)}\right] = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

# Alternative Approach

- Consider Voltage applied in coordinate axis
  - Voltage in z-direction independent of Voltage applied in x/y-direction
  - Haversian/Volkmann canals in parallel with Bone matrix

- Derive expression for effective resistivity in Vertical and Horizontal Directions

- Construct Conductivity Tensor **k** describing the effective conductivity of bone in the form:

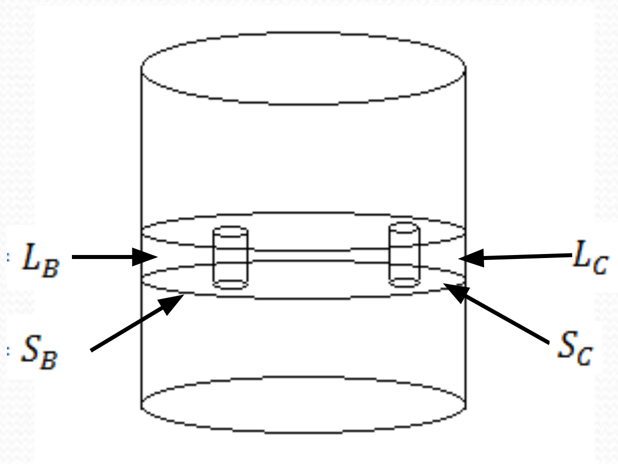
$$\begin{bmatrix} k_0 & 0 & 0 \\ 0 & k_0 & 0 \\ 0 & 0 & k_1 \end{bmatrix}$$

# Vertical Conductivity

- Consider Voltage applied in z-direction

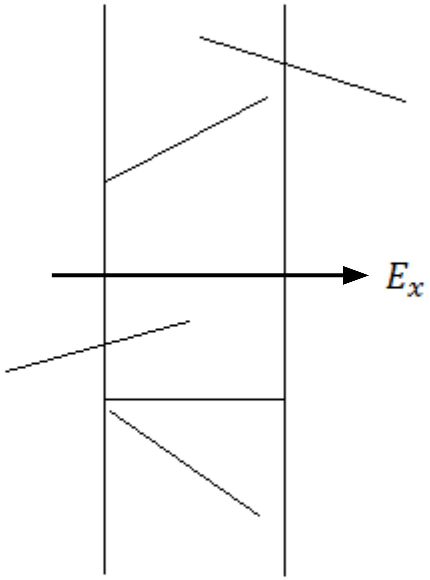
- Describe effective resistivity of Haversian canals and Bone Matrix as:

$$R = \frac{\rho_C \rho_B L}{\rho_C S_B + \rho_B s_C} \longrightarrow \rho_o = \frac{S_C}{\kappa} \frac{\rho_C \rho_B}{\rho_C S_B + \rho_B S_C}$$



# Horizontal Conductivity

- Consider Voltage applied in x/y-direction
- Projected Length of Volkmann canals not always same
  - Use probability distribution function to describe average effective length.




$$x(\alpha) = l\cos(\alpha) \qquad \alpha(x) = \arccos\left(\frac{x}{l}\right) \qquad p(\alpha) = \frac{1}{\pi}$$

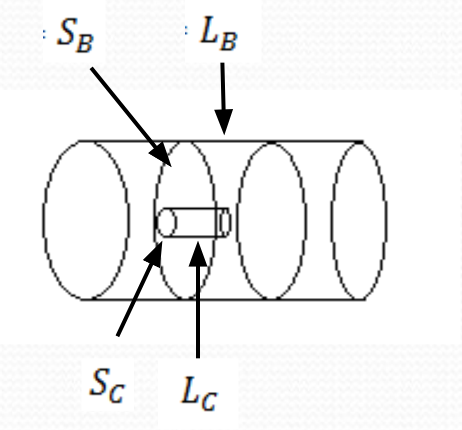
$$R(x)dx = p(\alpha)d\alpha \longrightarrow R(x) = p(\alpha)\frac{d\alpha}{dx} = \frac{-1}{\pi l\sqrt{1-\left(\frac{x}{l}\right)^2}}$$

$$\int_0^l R(x)x\,dx = \frac{-1}{\pi l}\int_0^l \frac{x}{\sqrt{1-\left(\frac{x}{l}\right)^2}}dx = \frac{2l}{\pi}$$

# Horizontal Conductivity (Cont)

- Use result from probability distribution function to use for length of Volkmann canal
- Describe effective resistivity of Volkmann canals and Bone Matrix as:

$$\rho_o = \frac{S_C}{\kappa} \frac{L_B \rho_C \rho_B}{L_C \rho_C S_B + L_B \rho_B S_C}$$



# Results

- Effective conductivity of the bone in vertical and horizontal directions are 0.013 and 0.034 $(\Omega m)^{-1}$, respectively
- Comparison of Values derived to values from paper

| | $A_1$ | $A_2$ |
|---|---|---|
| Single prolate spheroid $\gamma = 120$ | −2.001 | $-3.211 \times 10^3$ |
| Single prolate spheroid $\gamma = 80$ | −2.001 | $-1.568 \times 10^3$ |
| Single oblate spheroid $\gamma = 0.2$ | −8.016 | 6.683 |
| | $\sum R_{11} = \sum R_{22}$ | $\sum R_{33}$ |
| Haversian canals | −2.001 | $-3.213 \times 10^3$ |
| Osteocyte lacunae | −4.674 | −8.016 |
| Canaliculi and Volkman's canals | $-7.86 \times 10^2$ | −2.001 |

- Conductivity tensor describing conductivity of bone can be defined as:

$$\begin{bmatrix} 0.034 & 0 & 0 \\ 0 & 0.034 & 0 \\ 0 & 0 & 0.013 \end{bmatrix}$$

# Future Work

Consideration for future work

- Finish the calculations and modeling of contribution to conductivity from overlap of canals.

# Works Cited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