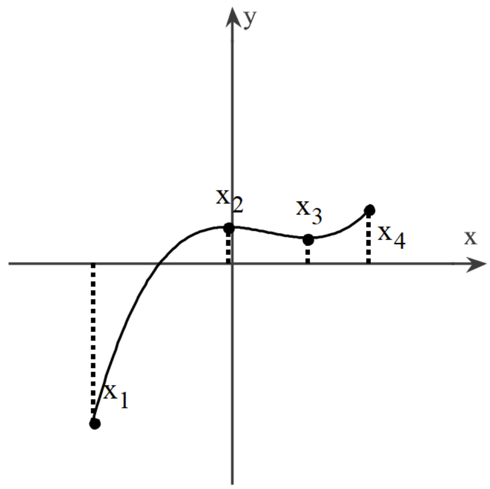
Math 116
Spring 2017

Study Guide for Exam 3 - Updated 4/7/17

1. For the graph below, identify each labeled point as an absolute maximum, absolute minimum, or neither.



(i) x_1 is

(A) an absolute maximum **(B)** an absolute minimum **(C)** neither

(ii) x_2 is

(A) an absolute maximum **(B)** an absolute minimum **(C)** neither

(iii) x_3 is

(A) an absolute maximum **(B)** an absolute minimum **(C)** neither

(iv) x_4 is

(A) an absolute maximum **(B)** an absolute minimum **(C)** neither

2. Find the value of x where the absolute minimum value of the function

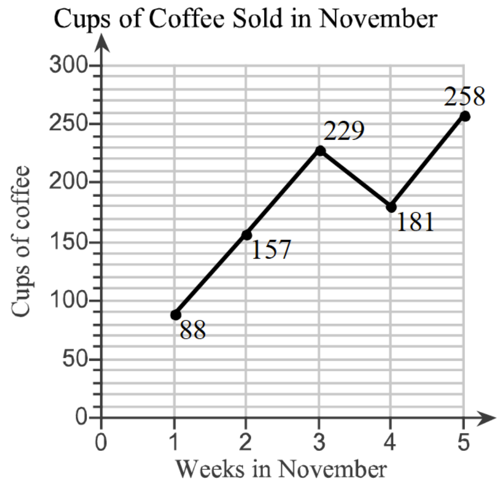
$$f(x) = x^3 + 2x^2 - 4x - 5$$

over the interval $[0, 2]$ occurs. (Round to two decimal place as needed.)

The absolute minimum occurs at

(A) $x = 0$ **(B)** $x = 0.67$ **(C)** $x = 1.79$ **(D)** $x = 2$ **(E)** none of these

3. The number of cups of coffee sold at Lauren's Coffee House for each week in November is shown in the graph below.



(i) Which of the following x-values yields an absolute minimum for the interval $[2, 5]$?

(A) $x = 2$ **(B)** $x = 3$ **(C)** $x = 4$ **(D)** $x = 5$ **(E)** None of these

(ii) Which of the following x-values yields an absolute minimum for the interval $(2, 5)$?

(A) $x = 2$ **(B)** $x = 3$ **(C)** $x = 4$ **(D)** $x = 5$ **(E)** None of these

(iii) Which of the following x-values yields a relative minimum for the interval $[2, 5]$?

(A) $x = 2.1$ **(B)** $x = 3$ **(C)** $x = 4$ **(D)** $x = 4.9$ **(E)** None of these

(iv) Which of the following x-values yields a relative minimum for the interval $(2, 5)$?

(A) $x = 2.1$ **(B)** $x = 3$ **(C)** $x = 4$ **(D)** $x = 4.9$ **(E)** None of these

4. The total profit $P(x)$ (in thousands of dollars) from the sale of x hundred thousand pillows is approximated by

$$P(x) = -x^3 + 12x^2 + 60x - 300, \text{ for } x \geq 5.$$

Find the number of pillows that must be sold to maximize profit.

The number of pillows is

(A) 10 **(B)** 500 **(C)** 1,000,000 **(D)** 500,000 **(E)** none of these

5. Find the minimum value of the *average cost* for the cost function

$$C(x) = x^3 + 35x + 250$$

over the interval $(0, 10]$.

The absolute minimum is

(A) between 95 and 105 **(B)** between 105 and 115 **(C)** between 115 and 125

(D) between 125 and 135 **(E)** between 135 and 145

6. A hotel has 290 units. All rooms are occupied when the hotel charges \$90 per day for a room. For every increase of x dollars in the daily room rate, there are x rooms vacant. Each occupied room costs \$34 per day to service and maintain. What should the hotel charge per day in order to maximize daily profit?

   (A) \$117 **(B)** \$173 **(C)** \$190 **(D)** \$207 **(E)** None of these

7. A baseball team is trying to determine what price to charge for tickets. At a price of \$18 per ticket, it averages 44,000 people per game. For every increase of \$1, it loses 1000 people. Every person at the game spends an average of \$8 on concessions. What price per ticket should be charged in order to maximize revenue?

   (A) \$27 **(B)** \$35 **(C)** \$13 **(D)** \$9 **(E)** None of these

8. Suppose that the cost function for a product is given by $C(x) = 0.003x^3 + 7x + 10,629$. Find the production level (i.e. value of x) that will produce the minimum average cost per unit $\overline{C}(x)$.

The production level is

(A) less than 25 **(B)** between 25 and 50 **(C)** between 50 and 75

(D) between 75 and 100 **(E)** more than 100

9. Consider the cost function $C(q) = 120 + 24q$ and the price function $p = 96 - 2q$. Find the number, q, of units that produces maximum profit.

The number of units that produces maximum profit is

(A) between 8 and 11 **(B)** between 11 and 14 **(C)** between 14 and 17

(D) between 17 and 20 **(E)** between 20 and 23

10. If the price charged for a candy bar is $p(x)$ cents, then x thousand candy bars will be sold in a certain city, where

$$p(x) = 164 - \frac{x}{10}.$$

(i) Find an expression for the total revenue from the sale of x thousand candy bars.

(ii) Find the value of x that leads to maximum revenue. (Note: to receive full credit, you must demonstrate that your answer yields an absolute maximum.)

(iii) Find the maximum revenue.

11. A local club is arranging a charter flight to Hawaii. The cost of the trip is \$560 each for 80 passengers, with a refund of \$5 per passenger for each passenger in excess of 80.

(i) Find the number of passengers that will maximize the revenue received from the flight. (Note: to receive full credit, you must demonstrate that your answer yields an absolute maximum.)

(ii) What is the maximum revenue?

12. Consider the cost function $C(q) = 80 + 19q$ and the price function $p = 67 - 2q$.

(i) Find an expression for profit from the sale of q items.

(ii) Find the number, q, of units that produces maximum profit. (Note: to receive full credit, you must demonstrate that your answer yields an absolute maximum.)

(iii) Find the maximum profit, P.

13. Consider the demand function

$$q = 402 - 0.3p^2$$

(i) Find the elasticity of demand for the demand function above for $p = \$22$.
The elasticity of demand for $p = \$22$ is

(A) between 0.0 and 0.5 **(B)** between 0.5 and 1.0 **(C)** between 1.0 and 2.0

(D) between 2.0 and 4.0 **(E)** between 4.0 and 6.0

FOLLOW-UP: *Be sure you can compute the elasticity of demand with all types of demand functions.*

(ii) Determine the value of q (if it exists) at which total revenue is maximized.
The value of q that maximizes revenue is

(A) between 215 and 230 **(B)** between 230 and 245 **(C)** between 245 and 260

(D) between 260 and 275 **(E)** does not exist

14. The short-term demand for crude oil in Country A in 2008 can be approximated by

$$q = f(p) = 1,615,368p^{-0.05},$$

where p represents the price of crude oil in dollars per barrel and q represents the per capita consumption of crude oil. Answer parts **(i)** and **(ii)**

(i) What is the elasticity of demand for oil when the price is $96 per barrel?
The elasticity of demand is

(A) less than 0.25 **(B)** between 0.25 and 0.75 **(C)** between 0.75 and 1.25

(D) between 1.25 and 1.75 **(E)** more than 1.75

(ii) Interpret the elasticity of demand. Choose the correct answer below.

(A) The demand is elastic, so as price increases, revenue decreases.

(B) The demand is elastic, so as price increases, revenue increases.

(C) The demand is inelastic, so as price increases, revenue increases.

(D) The demand is inelastic, so as price increases, revenue decreases.

15. Evaluate the following indefinite integral.

$$\int \left(e^{0.2x} + 3x^2\right) dx$$

(A) $5e^{0.2x} + x^3 + C$ **(B)** $0.2e^{0.2x} + x^3 + C$ **(C)** $5e^{0.2x} + 6x + C$

(D) $0.2e^{0.2x} + 6x + C$ **(E)** None of these

16. Evaluate the following indefinite integral.

$$\int \left(\frac{3}{\sqrt{x}} - \frac{1}{x}\right) dx$$

(A) $\frac{3}{2}\sqrt{x} - \ln|x| + C$ **(B)** $\frac{-3}{2\sqrt{x^3}} + \frac{1}{x^2} + C$ **(C)** $6\sqrt{x} - \ln|x| + C$

(D) $\frac{-3}{2\sqrt{x^3}} - 1 + C$ **(E)** None of these

17. Evaluate the following indefinite integral.

$$\int \left(10x^3 + 4x^2 - 3x + 5\right) dx$$

(A) $30x^2 + 8x - 3 + C$ **(B)** $\frac{5}{2}x^4 + \frac{4}{3}x^3 - \frac{3}{2}x^2 + 5x + C$ **(C)** $\frac{10}{3}x^4 + 2x^3 - 3x^2 + 5x + C$

(D) $\frac{10}{3}x^2 + 2x - 3 + C$ **(E)** None of these

18. Evaluate the following indefinite integral.

$$\int \left(\frac{e}{x^6} + \frac{e}{\sqrt{x}}\right) dx$$

19. Evaluate the following indefinite integral.

$$\int 6e^{-0.4x}dx$$

20. Find the cost function if the marginal cost function is

$$MC(x) = 6x - 3$$

and the fixed cost is \$11.

(A) $C(x) = 3x^2 - 3x + 11$ **(B)** $C(x) = 6x + 8$ **(C)** $C(x) = 11x$

(D) $C(x) = 3x^2 + 8x$ **(E)** None of these

21. Find the revenue function if the marginal revenue function is

$$MR(x) = 6x^2 + 4x + 5.$$

(A) $R(x) = 6x^2 + 4x + 5$ **(B)** $R(x) = 6x^3 + 4x^2 + 5x$ **(C)** $R(x) = 12x + 4$

(D) $R(x) = 2x^3 + 2x^2 + 5x$ **(E)** None of these

22. Find the demand function for the marginal revenue function.

$$MR(x) = 0.03x^2 - 0.04x + 208$$

(A) $p(x) = 0.01x^3 - 0.02x^2 + 208x$ **(B)** $p(x) = 0.03x - 0.04 + \dfrac{208}{x}$

(C) $p(x) = 0.01x^2 - 0.02x + 208$ **(D)** $p(x) = 0.03x^2 - 0.04x + 208$

(E) None of these

23. The approximate rate of change in the number (in billions) of monthly text messages is given by the equation

$$f'(t) = 4.2t - 7.4$$

where t represents the number of years since 2000. In 2000 ($t = 0$) there were approximately 9.2 billion monthly text messages. How many monthly text messages were there in 2005?

In 2005, the number of text messages was

(A) less than 10 billion

(B) between 10 billion and 20 billion

(C) between 20 billion and 30 billion

(D) between 30 billion and 40 billion

(E) more than 40 billion

24. The marginal profit in dollars on Brie cheese sold at a cheese store is given by

$$P'(x) = x(90x^2 + 60x),$$

where x is the amount of cheese sold, in hundreds of pounds. The "profit" is $-\$20$ when no cheese is sold. Find the profit from selling 200 pounds of Brie cheese.

The profit is

(A) less than \$550

(B) between \$550 and \$650

(C) between \$650 and \$750

(D) between \$750 and \$850

(E) more than \$850

25. Which of the following substitutions would be the most useful in determining the indefinite integral:

$$\int 3x^2 e^{x^3+2} dx$$

(A) $u = 3x^2$ **(B)** $u = x^3 + 2$ **(C)** $u = 3$ **(D)** $u = x$ **(E)** None of these

26. Which of the following substitutions would be the most useful in determining the indefinite integral:

$$\int \frac{2x}{x^2+4} dx$$

(A) $u = 2x$ **(B)** $u = \dfrac{x}{x^2+4}$ **(C)** $u = x^2 + 4$ **(D)** $u = x$ **(E)** None of these

27. If we use the substitution $u = x^2 + 11$, then which of the following is equivalent to

$$\int x(x^2 + 11)^{1/3} dx.$$

(A) $\int u^{1/3} du$ **(B)** $\int \frac{u^{1/3}}{2} du$ **(C)** $\int (u - 11)^{1/2} u^{1/3} du$

(D) $\int \frac{(u-11)^{1/2} u^{1/3}}{2} du$ **(E)** None of these

28. Find the indefinite integral.

$$\int \frac{6}{(6x+1)^2}\,dx$$

29. Evaluate the following indefinite integral.

$$\int \frac{2x}{4+x^2}\,dx$$

(A) $\ln|4+x^2|+C$

(B) $\dfrac{x^2}{4x+\frac{1}{3}x^3}+C$

(C) $\dfrac{1}{4}x^2+2\ln|x|+C$

(D) $x^2\ln|4+x^2|+C$

(E) None of these

30. Evaluate the following indefinite integral.

$$\int \frac{4e^{4x}}{2+e^{4x}}dx$$

(A) $\dfrac{e^{4x}}{2x+0.25e^{4x}}+C$ **(B)** $\ln|2+e^{4x}|+C$ **(C)** $4\ln|2+e^{4x}|-2-e^{4x}+C$

(D) $\dfrac{1}{2}e^{4x}+4x+C$ **(E)** None of these

31. If we use the substitution $u = 2x + e^{-5x}$, then which of the following is equivalent to

$$\int \frac{2-5e^{-5x}}{(2x+e^{-5x})^2}dx.$$

(A) $\displaystyle\int \frac{du}{u}$ **(B)** $\displaystyle\int \frac{u^2}{du}$ **(C)** $\displaystyle\int \frac{du}{u^2}$ **(D)** $\displaystyle\int \frac{u}{du}$ **(E)** None of these

32. Approximate the area under the graph of $f(x) = e^x + 1$ and above the x-axis from $x = -3$ to $x = 3$ with rectangles with $n = 3$. Answer parts **(i)** and **(ii)**

(i) Approximate the area under the graph of $f(x) = e^x + 1$ and above the x-axis from $x = -3$ to $x = 3$ with rectangles, using left endpoints, with $n = 3$. The approximate area is

(A) less than 15 **(B)** between 15 and 25 **(C)** between 25 and 35

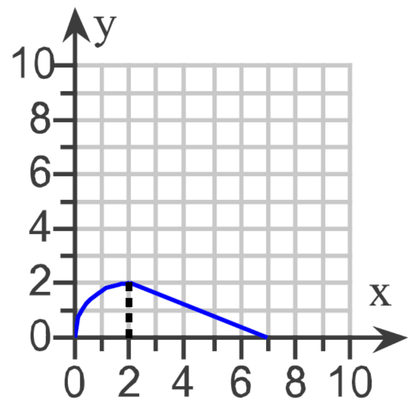
(D) between 35 and 45 **(E)** more than 45

(ii) Approximate the area under the graph of $f(x) = e^x + 1$ and above the x-axis from $x = -3$ to $x = 3$ with rectangles, using right endpoints, with $n = 3$. The approximate area is

(A) less than 15 **(B)** between 15 and 25 **(C)** between 25 and 35

(D) between 35 and 45 **(E)** more than 45

33. Find $\int_0^7 f(x)dx$ for the graph of $y = f(x)$ below, where $f(x)$ consists of line segments and circular arcs.



The value of $\int_0^7 f(x)dx$ is

(A) between 7.6 and 7.8 **(B)** between 7.8 and 8.0 **(C)** between 8.0 and 8.2

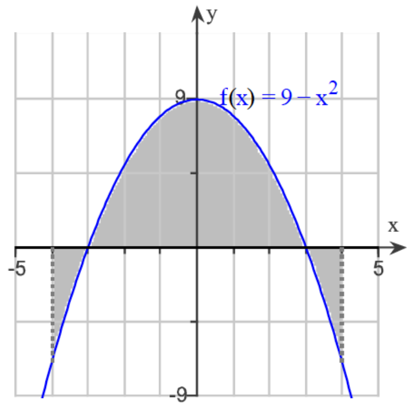
(D) between 8.2 and 8.4 **(E)** between 8.4 and 8.6

34. Find the exact value of the integral using formulas from geometry.

$$\int_0^6 \sqrt{36 - x^2}dx$$

(A) 36π **(B)** 9π **(C)** 18π **(D)** 6π **(E)** None of these

35. Find the total area of the shaded regions.



The total area of the shaded regions is

(A) between 30 and 35 **(B)** between 35 and 40 **(C)** between 40 and 45

(D) between 45 and 50 **(E)** between 50 and 55

36. Find the area under the graph of $f(x) = -x^2 + 5$ and above the x-axis from $x = -2$ to $x = 2$. The area is

(A) less than 4 **(B)** between 4 and 8 **(C)** between 8 and 12

(D) between 12 and 16 **(E)** more than 16

37. Evaluate the definite integral

$$\int_2^5 \left(e^{-0.2x} + \frac{1}{x}\right) dx$$

The definite integral is

(A) between -1 and 0 **(B)** between 0 and 1 **(C)** between 1 and 2

(D) between 2 and 3 **(E)** between 3 and 4

38. If we use the substitution $u = 1 - x$, then which of the following is equivalent to

$$\int_{-2}^{1} \sqrt{1-x}dx.$$

(A) $-\int_{-2}^{1} \sqrt{u}du$ **(B)** $\int_{-2}^{1} \sqrt{u}du$ **(C)** $-\int_{0}^{3} \sqrt{u}du$

(D) $\int_{0}^{3} \sqrt{u}du$ **(E)** None of these

39. Evaluate the definite integral

$$\int_4^5 \frac{3}{(3x+2)^2}\,dx$$

40. Using the Fundamental Theorem of Calculus, which of the following expressions is equivalent to

$$\int_2^7 \left(e^x + x^5\right) dx$$

(A) $\left[e^7 + 5(7)^4\right] - \left[e^2 + 5(2)^4\right]$

(B) $\left[\frac{e^8}{8} + \frac{(7)^6}{6}\right] - \left[\frac{e^3}{3} + \frac{(2)^6}{6}\right]$

(C) $\left[e^7 + \frac{(7)^6}{6}\right] - \left[e^2 + \frac{(2)^6}{6}\right]$

(D) $\left[7e^6 + 5(7)^4\right] - \left[2e^1 + 5(2)^4\right]$

(E) None of these

41. A small company of science writers found that its rate of profit (in dollars) after t years of operation is given by the function below.

$$P'(t) = 900(t+1)\left(t^2 + 2t + 2\right)^{\frac{1}{3}}.$$

Find the total profit in the first three years. The total profit is

(A) less than \$7,000

(B) between \$7,000 and \$9,000

(C) between \$9,000 and \$11,000

(D) between \$11,000 and \$13,000

(E) more than \$13,000

42. Find the area between the curves

$$x = -5, \qquad x = 3, \qquad y = 2x \qquad y = x^2 - 3$$

The area between the curves is

(A) less than 30 **(B)** between 30 and 40 **(C)** between 40 and 50

(D) between 50 and 60 **(E)** more than 60

43. Find the producers' surplus if the supply function for pork bellies is given by

$$S(q) = q^{5/2} + 2q^{3/2} + 54.$$

Assume the supply and demand are in equilibrium at $q = 16$. The producers' surplus is

(A) between \$11,000 and \$11,500 **(B)** between \$11,500 and \$12,000

(C) between \$12,000 and \$12,500 **(D)** between \$12,500 and \$13,000

(E) between \$13,000 and \$13,500

44. Find the consumers' surplus if the demand function for a particular beverage is given by

$$D(q) = \frac{4000}{(4q+1)^2}.$$

Assume the supply and demand are in equilibrium at $q = 3$. The consumers' surplus is

(A) less than \$800 **(B)** between \$800 and \$840

(C) between \$840 and \$880 **(D)** between \$880 and \$920

(E) more than \$920

45. The supply function for oil is given (in dollars) by $S(q)$, and the demand function is given (in dollars) by $D(q)$

$$S(q) = q^2 + 13q; \qquad D(q) = 1054 - 15q - q^2.$$

(i) Graph the supply and demand curves on the same axes.

(ii) Find the point at which supply and demand are in equilibrium.

(iii) Find the consumers' surplus.

(iv) Find the producers' surplus.

46. The function $f(x) = 1000x - 100x^2$ represents the rate of flow of money in dollars per year. Assume a 10-year period at 4% compounded continuously.

(i) Find the present value of the money flow at the end of 10 years.

(ii) Find the accumulated amount of money flow at the end of 10 years.

47. Find the present value of a continuous stream of income over 6 years when the rate of income is constant at \$33,000 per year and the interest rate is 4% compounded continuously.

The present value is

(A) less than \$175,000

(B) between \$175,000 and \$180,000

(C) between \$180,000 and \$185,000

(D) between \$185,000 and \$190,000

(E) more than \$190,000

48. The rate of a continuous money flow starts at \$1100 and increases exponentially at 3% per year for 5 years. Find the present value if interest earned is 4% compounded continuously.

The present value is

(A) between \$4600 and \$5000

(B) between \$5000 and \$5400

(C) between \$5400 and \$5800

(D) between \$5800 and \$6200

(E) between \$6200 and \$6600

49. The rate of a continuous money flow starts at \$1100 and increases exponentially at 3% per year for 5 years. Find the accumulated amount of money flow at the end of 5 years if interest earned is 4% compounded continuously.

The accumulated amount is

(A) less than \$5000

(B) between \$5000 and \$5400

(C) between \$5400 and \$5800

(D) between \$5800 and \$6200

(E) more than \$6200

Formulas You Might Find Useful

$$I = Prt$$

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$A = Pe^{rt}$$

$$r_E = \left(1 + \frac{r}{m}\right)^{m} - 1$$

$$r_E = e^r - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}\left[u(x) \cdot v(x)\right] = u(x) \cdot v'(x) + v(x) \cdot u'(x)$$

$$\frac{d}{dx}\left[\frac{u(x)}{v(x)}\right] = \frac{v(x) \cdot u'(x) - u(x) \cdot v'(x)}{[v(x)]^2}$$

$$\frac{d}{dx}\left[a^x\right] = (\ln a)a^x$$

$$\frac{d}{dx}\left[\log_a(x)\right] = \frac{1}{(\ln a)x}$$

$$E = -\frac{p}{q} \cdot \frac{dq}{dp}$$

$$q = \sqrt{\frac{2fM}{k}}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int a^{kx} dx = \frac{a^{kx}}{k(\ln(a))} + C$$

$$\int x^{-1} dx = \ln|x| + C$$

$$C.S. = \int_0^{q_0} \left(D(q) - p_0\right) dq$$

$$P.S. = \int_0^{q_0} \left(p_0 - S(q)\right) dq$$

$$P = \int_0^T f(t)e^{-rt} dt$$

$$A = e^{rT} \int_0^T f(t)e^{-rt} dt$$

Solutions to Study Guide - Part 3

1. **(i)** B
   (ii) C
   (iii) C
   (iv) A
2. B
3. **(i)** A
   (ii) E
   (iii) C
   (iv) C
4. C
5. B
6. D
7. A
8. E
9. D
10. **(i)** $R(x) = \left(164 - \dfrac{x}{10}\right)x = 164x - \dfrac{x^2}{10}$
    (ii) $x = 820$
    (iii) \$672,400
11. **(i)** 96
    (ii) \$46,080
12. **(i)** $P = (67 - 2q)q - (80 + 19q) = -2q^2 + 49q - 80$
    (ii) $q = 12$
    (iii) $P = 208$
13. **(i)** C
    (ii) D
14. **(i)** A
    (ii) C
15. A
16. C
17. B
18. $\dfrac{ex^{-5}}{-5} + \dfrac{ex^{1/2}}{1/2} + C = -\dfrac{e}{5x^5} + 2e\sqrt{x} + C$
19. $\dfrac{6e^{-0.4x}}{-0.4} + C = -15e^{-0.4x} + C$

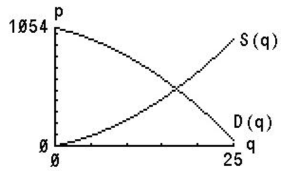
20. A
21. D
22. C
23. C
24. A
25. B
26. C
27. B
28. Let $u = 6x + 1$, so $du = 6dx$ and

$$\int \frac{6}{(6x+1)^2} dx = \int u^{-2} du = -u^{-1} + C = -\frac{1}{6x+1} + C.$$

29. A
30. B
31. C
32. **(i)** A
    (ii) E
33. C
34. B
35. C
36. D
37. D
38. D
39. Let $u = 3x + 2$, so $du = 3dx$

$$\int_4^5 \frac{3}{(3x+2)^2} dx = \int_{14}^{17} \frac{du}{(u)^2} = -u^{-1}\big|_{14}^{17} = -\frac{1}{17} + \frac{1}{14} = \frac{3}{238}$$

40. C
41. E
42. E
43. D
44. C

45. **(i)** 

(ii) $q = 17$; $p = 510$

(iii) \$5442.83

(iv) \$5153.83

46. **(i)** \$13,700.17

(ii) \$20,438.26

47. B

48. B

49. E