Critical Pts Worksheet soln

2A) $f(x) = x^4 + 2x^3 - 1$

i) Find crit. pts:

$$f'(x) = 4x^3 + 6x^2$$

Crit pts where $f' = 0$ or undefined
↳ nowhere

$$4x^3 + 6x^2 = 0$$

$$x^2(4x+6) = 0$$

$$x = 0, \quad x = -6/4 = -3/2$$

So the critical pts are $\boxed{x = 0,\ x = -3/2}$

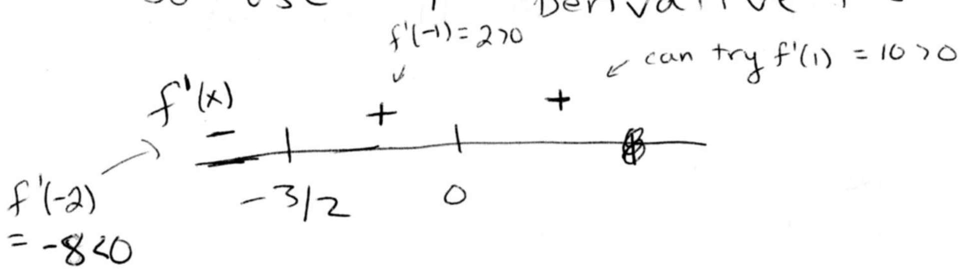
ii) Find local extrema:

2nd Derivative test:

$$f''(x) = 12x^2 + 12x$$

$f''(0) = 0 \rightarrow$ 2nd derivative test inconclusive

So use 1st Derivative test!



So f has a local min when $x = -3/2$

(no local extrema at $x = 0$)

iii) Find Inflection Points:

$f''(x) = 12x^2 + 12x$

Possible Infl. pts are where $f'' = 0$ or undefined

↳ nowhere

$12x^2 + 12x = 0$

$x(12x + 12) = 0$

$x = 0,\ x = -1$

Now test whether $x = 0, -1$ actually are inf. pts:

$f''(-2) > 0$ $\quad f''(-\tfrac{1}{2}) < 0$

$f''(x)$: $+$ $\quad -1 \quad$ $-$ $\quad 0 \quad$ $+$ $\quad \leftarrow f''(1) > 0$

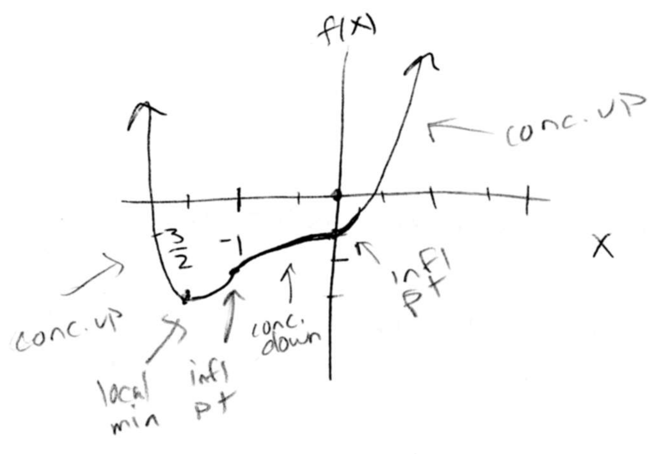
So since f changes concavity at $x = -1, 0$

These are both inflection points.

iv) $f(x)$ is conc. down on $(-1, 0)$

and conc. up on $(-\infty, -1) \cup (0, \infty)$

v)



B) $f(x) = \frac{8x-16}{x^2}$

$= 8x^{-1} - 16x^{-2}$

i) crit pts.

$f'(x) = -8x^{-2} + 32x^{-3}$

$= \frac{-8x+32}{x^3}$

crit pts where

$f' = 0$ or undef

$\downarrow$ $\hookrightarrow x = 0$ not in domain!

$-8x + 32 = 0$ (So $x=0$ NOT crit. pt)

$32 = 8x$

$x = 4$

So $\boxed{x=4}$ is the only critical point.

ii) local extrema

2nd deriv. test.

$f''(x) = 16x^{-3} - 96x^{-4}$

$= \frac{16x - 96}{x^4}$

$f''(4) = \frac{16 \cdot 4 - 96}{4^4} = \frac{64-96}{4^4} < 0$

$\Rightarrow$ f has a local max when $x=4$

iii) Inflection Pts:

possible infl. pts when $f''=0$ or undef

↳ $x=0$, but not in domain

$16x-96=0$

$x=\frac{96}{16}=6$

So $x=6$ is the only possible infl. pt.

to check, see if f changes concavity at $x=6$:

$$\begin{array}{c|c} f''(x) \quad - & + \\ \hline 6 & \end{array}$$

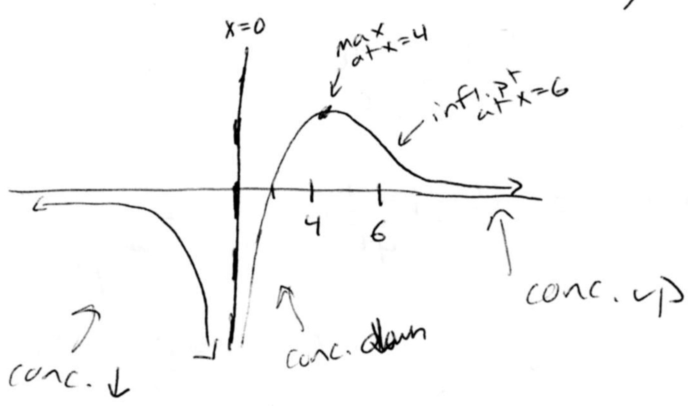
So f changes conc. at $x=6$

$\Rightarrow$ $\boxed{x=6 \text{ is an inflection pt}}$

iv)

From ↑, we see $f(x)$ is conc. down on $(-\infty,6)$ and conc. up on $(6,\infty)$

v) $f(x)$ has vert. asympt. on $x=0$, horiz asympt on $y=0$



c) $f(x) = 2x + 3x^{2/3}$

i) <u>crit pts</u>

$$f'(x) = 2 + 2x^{-1/3}$$
$$= 2 + \frac{2}{x^{1/3}}$$

crit. pts when $f' = 0$ or undefined

$\swarrow$ $\qquad\qquad$ $\hookrightarrow x = 0$

$$2 = -\frac{2}{x^{1/3}}$$

$$x^{1/3} = -1$$
$$x = (-1)^3$$
$$x = -1$$

So critical pts at $\boxed{x = -1 \text{ and } x = 0}$

ii) <u>local extrema</u>:

1st derivative test (since $f''(0)$ undefined, use 1st derv test instead of 2nd derv test)

$f'(x)$: $+$ on $(-\infty, -1)$, $-$ on $(-1, 0)$, $+$ on $(0, \infty)$

So f has a local max when $x = -1$ and a local min when $x = 0$.

iii) Inflection Pts

possible inf. pts when $f''=0$ or undef.

$f''(x) = -\frac{2}{3}x^{-4/3}$

$-\frac{2}{3}x^{-4/3}$ is never 0!

But $-\frac{2}{3}x^{-4/3}$ is undef. when $x=0$

$$= \frac{-2}{3x^{4/3}}$$

Now check if $x=0$ is infl. pt:

$f''(x)$ $-$ | $-$
0

f'' is conc down on $(-\infty,0)$ and $(0,\infty)$

so f doesn't change concavity.

$\Rightarrow$ f has no inflection pts

iv) f is conc. down on $(-\infty,0)\cup(0,\infty)$ and never conc. up. (I would also accept $(-\infty,\infty)$ in this case)

v)