Analysis of Human Cardiogram using Embedology

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Human Cardiogram[2]

- The goal is to try to model human cardiogram by using embedding techniques
- The data is from a sample of electrocardiogram recording acquired from MIT Polysomnographic database will be used to model it using embedding techniques
- Before we use the embedding techniques on human cardiogram, we first look into how Embedology works.

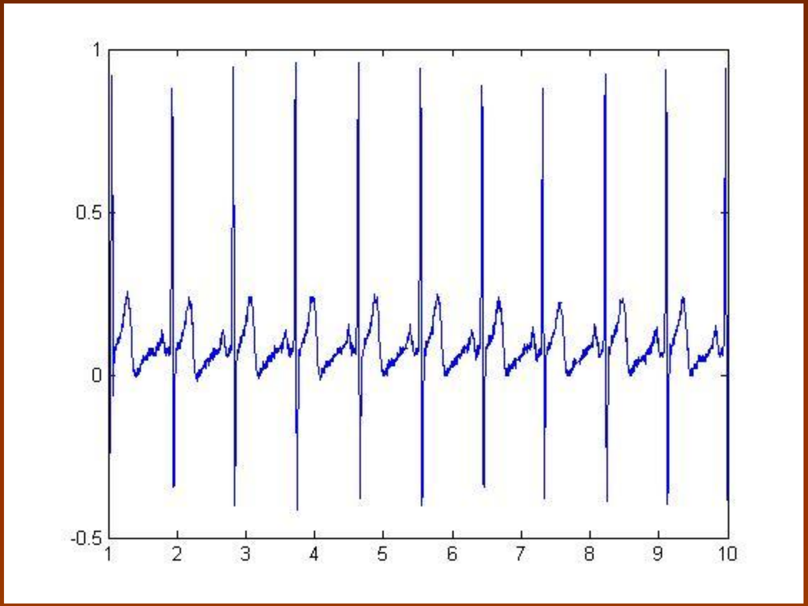

Figure 1: First 10 seconds of a heartbeat reproduced by MATLAB

- [2] M. Perc, _Nonlinear time series analysis of the human electrocardiogram_, Eur. J. Phys. **26**, 757-768 (2005).

1. Introduction of Embedology[1]

- Embedology is the study of embedding techniques used to embed data of one form to another
- The paper first highlights two basic theorems that are important when it comes to embedding data:
  - Fractal Whitney Embedding Theorem
  - Fractal Delay Embedding Theorem
- These two theorems are crucial in determining how to embed the electrocardiogram data acquired from MIT.

[1] Tim Sauer, James A. Yorke, and Martin Casdagli , *Embedology* J. Stat Phys **Vol 65**, 579-616 (1991).

2.1 Fractal Whitney Embedding Prevalence Theorem

- Fractal Whitney Embedding Prevalence Theorem introduces two concepts that are necessary for this theorem to work, one is that the data has to be dense and the other is that is has to be open
- Embeddings that are dense refers to the embedding near a set
- Embedding that are open refers to the fact that small perturbations in a smooth map will still be embeddings
- Box dimension is similar to a fractal dimension and is found through the following equation: $\mathrm{boxdim}(A) = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{-\log \varepsilon}$
- In order to prove that every map is an embedding the paper proposed Definition 2.1 which states:
  - A Borel subset S of a normed linear space V is prevalant if there is a fine-dimensional subspace E of V such that for each *v* in V, *v* + *e* belongs to S for almost every *e* in E.
- From the above definition the paper proposes the prevalance version of Whitney embedding theorem:
  - Let A be a compact smooth manifold of dimension *d* contained in R^k. Almost every smooth map $R^k \to R^{2d+1}$ is an embedding of A

2.1 Fractal Whitney Embedding Prevalence Theorem

- The main premise of this theorem is to prove that in order for a function to be smooth:
  - F is a one-to-one on A
  - F is an immersion on each compact subset C of a smooth manifold contained in A
- The above is true only when n > 2 * boxdim(A), when a function is transformed from R^k to R^n

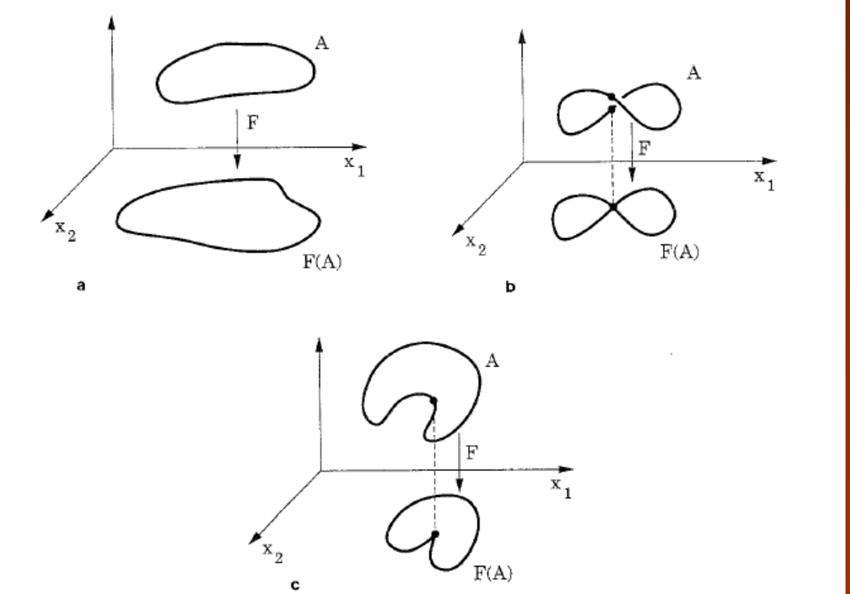

Fig. 1. (a) An embedding F of the smooth manifold A into R^2. (b) An immersion that fails to be one-to-one. (c) A one-to-one map that fails to be an immersion.

Figure 2: reproduced from reference [1], Page 583 (1991)

Fractal Delay Embedding Prevalence Theorem

- From the Fractal Whitney Prevalence Theorem, one can expand this theorem to delay coordinates
- Theorem introduces delay-coordinate maps, where a function with multiple independent variables can be transformed into a function with one variable
- A delay coordinate map is defined as:

$$F(h, \Phi, T)(x) = (h(x), h(\Phi_{-T}(x)), h(\Phi_{-2T}(x)), \ldots, h(\Phi_{-(n-1)T}(x)))$$

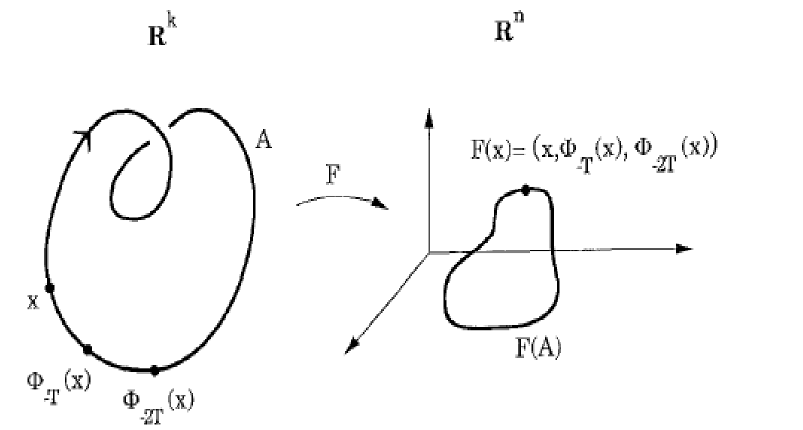

Fig. 2. The attractor on the left is mapped using delay coordinates into the reconstruction space on the right.

Figure 3: reproduced from reference [1], Page 588 (1991)

Fractal Delay Embedding Prevalence Theorem

- Combining the Fractal Whitney Embedding Prevalence Theorem to Delay coordinates one can say that for every smooth function, the delay coordinate map F(h,Φ, T) is:
  - One-to-one on A
  - An immersion of a smooth manifold is contained in A
- For the above statement to be true, there should be no periodic orbits, and the linearization should have distinct eigenvalues.

Applying Embedding techniques to Human Cardiogram

- Using the delay coordinate map technique defined by Takens from slide 6:

$$F(h, \Phi, T)(x) = (h(x), h(\Phi_{-T}(x)), h(\Phi_{-2T}(x)), \ldots, h(\Phi_{-(n-1)T}(x)))$$

- We plot each of the point in 3 dimension.
- Maps are shown to the right with different delays.
- From trying out different delays, the ideal delay is when delay = 9

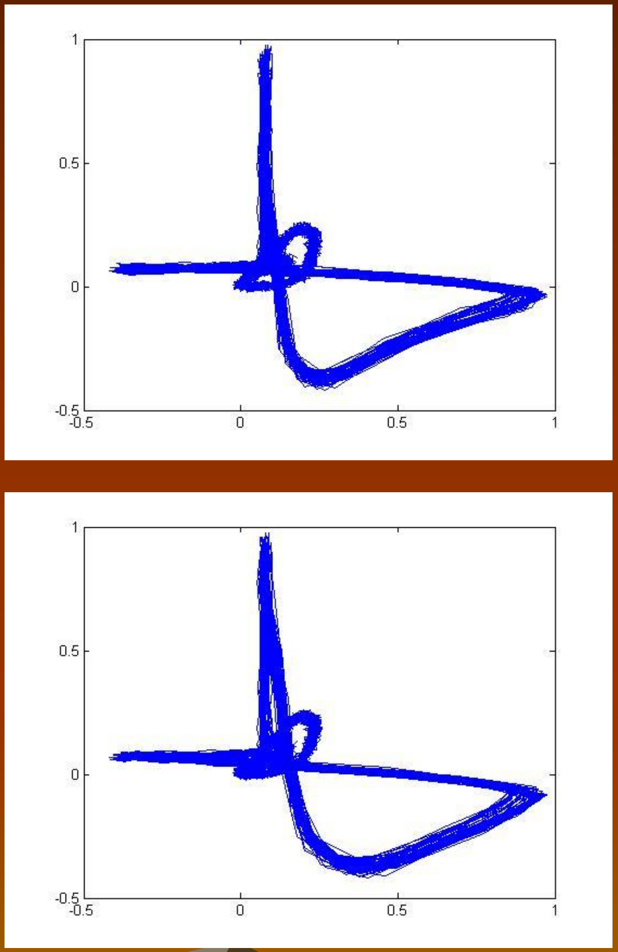

Figure 4: reproduced by MATLAB

Summary

- Human electrocardiogram can be modeled with two important theorems from Embedology
- From Fractal Whitney Embedding Prevalence theorem one can apply it to delay coordinates.
- Since the data acquired fits the requirements listed in Fractal Whitney Embedding, we can embed it using delay coordinates
- Embedology is a powerful tool and is useful in modeling Human electrocardiogram and understanding the dynamics of the human heart

Sources

1. Tim Sauer, James A. Yorke, and Martin Casdagli , *Embedology* J. Stat Phys **Vol 65**, 579-616 (1991)
2. M. Perc, *Nonlinear time series analysis of the human electrocardiogram*, Eur. J. Phys. **26**, 757-768 (2005).

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