# MATH 583 B MIDTERM

March 4, 1999

You have 75 minutes to solve four problems (each of them is worth 25 points). To receive full credit, you must justify your steps with a theorem or a reason. Good luck !

1. For $\lambda$ a real number, solve the following integral equations:

   (a) $\varphi(x) = e^x + \lambda \int_0^x e^{x-\xi}\ \varphi(\xi)\ d\xi, \qquad 0 \le x \le 1$

   (b) $\psi(x) = e^x + \lambda \int_0^1 e^{x-\xi}\ \psi(\xi)\ d\xi, \qquad 0 \le x \le 1$

2. Find the Neumann iterates for the integral equation

$$u(x) = e^x + \int_0^x e^{x-\xi}\ u(\xi)\ d\xi$$

3. (a) Find the Fourier transform, $\hat{f}(k) = \int_{-\infty}^{\infty} f(x) e^{-ikx}\ dx$, of $f(x) = \dfrac{e^{-|x|}}{2}(1+|x|)$.

   (b) Use Fourier transforms to solve $\left(\dfrac{d^2}{dx^2} - 1\right)^2 u = h(x)$. Your answer should be in terms of $h$.

   (c) Use the above to write an integral equation for the solution to

$$\frac{\partial u}{\partial t} = \left(\frac{\partial^2}{\partial x^2} - 1\right)^2 u, \qquad \lim_{|x|\to+\infty} u(x) = 0.$$

4. (a) Find the eigenfunctions and eigenvalues of the integral operator

$$A \cdot = \int_0^{2\pi} \cos(x+y)\ \cdot\ dy$$

   (b) Find $A^*$, the adjoint of $A$.

   (c) What can you say about the dimension of $Ker(A^*)$ ? Find a function in $Ker(A^*)$.

   (d) Find a function $f$ such that the equation $Au = f$ has no solution.