MATH 583 B PROJECT

February 4, 1999

The goal of this project is to understand the concepts of convective and absolute instabilities in open flows. As a basis for our investigation, we will read part of the following paper: L. Brevdo and T.J. Bridges, *Absolute and convective instabilities of spatially periodic flows*, Phil. Trans. Roy. Soc. London **A 354**, 1027 (1996). A copy of this paper was scanned by the Library staff and put on reserve in pdf format. It is available from the course web page at www.math.arizona.edu/∼lega/583/Spring99/583b.html (click on Reserve Materials and enter user name and password).

This paper is not an easy one and you should not get discouraged after having browsed through it. You will see that a lot can be done without resorting to the heavy machinery of Floquet analysis used in the paper. In fact, you will not need this at all. You will however need to use the material covered in the first part of the Methods course, namely contour integration, Green's functions, Fourier and Laplace transforms, as well as techniques for the linear stability analysis of solutions to partial differential equations.

Your final report on this project should be typed and should include the following:

1. A description in simple terms of the notions of convective and absolute instabilities, as well as a brief discussion of when and where such concepts are relevant.

2. A discussion of the formal mathematical analysis of these instabilities, either in terms of Green's functions (see reference to Huerre and

Monkewitz's work in the paper), or in terms of Laplace and Fourier transform, as done in the paper.

3. An application to the linear stability analysis of plane wave solutions of the complex Ginzburg-Landau equation

$$\frac{\partial \psi}{\partial t} = \psi + (1 + ia)\frac{\partial^2 \psi}{\partial x^2} - (1 - ib)|\psi|^2 \psi.$$

4. Conclusions, open questions or interesting ideas that you might have had while working on this problem.

You should try to work together and discuss your results amongst yourselves. You should also take advantage of each other's areas of expertise (some of you may be better at performing calculations; others may prefer using a computer to develop some intuition for the problem). It is crucial that you learn how to share your results, compare them to what others have found, and that you use this exchange of ideas to build a consistent understanding of the phenomenon under study. The following is a series of suggestions meant to guide you in the analysis of the notions of convective and absolute instability. Whether you decide to follow all or part of these guidelines is entirely up to you.

1. Read pages 2 and 3 of the paper.

   (a) Use techniques seen in class to find the solution (see Eqn. (1.8)) to the partial differential equation

   $$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + a\frac{\partial u}{\partial x} + u, \qquad -\infty < x < \infty,\ t > 0$$

   with initial condition $u(x, 0) = \exp(-bx^2/4)$.

   (b) Discuss the concepts of convective and absolute instabilities on the solution found above.

   (c) Can you find the solution to this equation corresponding to a general initial condition ?

   (d) Using a Fourier mode expansion for solutions to linear equations, as discussed in class last semester, what can you say about the stability of $u(x, t) = 0$? Does this contradict the result of (b) above ?

(e) Draw a space-time diagram showing the evolution of $u(x,t)$ given by (1.8) for $|a| < 2$ and $|a| > 2$. Use these diagrams to explain the difference between absolute and convective instabilities.

(f) You may do a numerical simulation of the partial differential equation given above to illustrate the behavior of the system for different initial conditions. If so, explain your results.

2. Read pages 4 and 5 of the paper.

   (a) Explain Eqns. (1.10) and (1.11).

   (b) Based on your understanding of Fourier and Laplace transforms, can you see why the pinching condition (see page 4, bottom) is related to the transition between convective and absolute instabilities ?

3. Apply the above analysis to the complex Ginzburg-Landau equation.

   (a) It is not necessary to use Floquet theory to analyze the linearized equation about a plane wave solution since a change of variable (remember our discussion of this problem last semester) turns the linearized system into two equations with constant coefficients. The analysis is therefore simpler than what is done in the paper.

   (b) Use a computer to describe the loci of the roots of the dispersion relation in the complex plane as the imaginary part σ of $\omega = \omega_r + i\sigma$ is decreased toward zero. Do you understand why $\omega(k)$ has a saddle point when two roots collide ? Why should these two roots come from opposite sides of the real axis in the complex k-plane in order to have an absolute instability ?

   (c) Compare your results to those discussed on pages 27-34.

Feel free to come by and discuss your results with me or ask for more detailed guidelines and/or for other articles and references on convective and absolute instabilities. I hope you will enjoy this experience.