Math 583 B
Homework Assignment # 8

Due April 1, 1999

Perturbation Expansions

1. Approximate the root of $x^3 - x^2 + 0.01 = 0$ near $x = 1$. What difficulty is encountered for the root near $x = 0$?

2. Approximate all roots of $\epsilon x^3 + x - 1 = 0$.
   Hint: To find large roots, set $x = 1/y$ and find y as a function of ϵ.

3. Verify the following order relations:

   (a) $\epsilon^2 \tanh(\epsilon) = O(\epsilon^2)$ as $\epsilon \to \infty$

   (b) $\dfrac{\sqrt{\epsilon}}{1 - \cos(\epsilon)} = O(\epsilon^{-3/2})$, as $\epsilon \to 0^+$

   (c) $\ln(\epsilon) = o(\epsilon^{-p})$ as $\epsilon \to 0^+$, for all $p > 0$.

   (d) $\displaystyle\int_0^{\epsilon} e^{-x^2}\, dx = O(\epsilon)$, as $\epsilon \to 0^+$

   (e) $e^{\tan(\epsilon)} = O(1)$, as $\epsilon \to 0$