Math 583 B
Homework Assignment # 7

Due March 25, 1999

Euler-Lagrange equations

1. Find and solve the Euler-Lagrange equations for the following functionals:

   (a) $F(y) = \int_a^b \frac{y_x^2}{x^3}\, dx, \qquad 0 < a < b.$

   (b) $F(y) = \int_a^b (y^2 + y_x^2 + 2ye^x)\, dx.$

   (c) $F(u) = \int\int_R \left[\frac{1}{2}(u_{xx}^2 + u_{yy}^2) + \mu u_{xx}u_{yy} + (1-\mu)u_{xy}^2\right] dx\, dy$, where μ is a specified constant and R some specified domain.

2. Find the extremal path for:

   (a) $F(y) = \int_1^2 \frac{\sqrt{1+y_x^2}}{x}\, dx$, subject to boundary conditions $y(1) = 0$ and $y(2) = 1$.

   (b) $F(y) = \int_1^e (\frac{1}{2}x^2y_x^2 - \frac{1}{8}y^2)\, dx$, with $y(1) = 1$ and $y(e)$ unspecified.

3. Extremize $F(y) = \int_0^1 (y_x^2 + x^2)\, dx$, with boundary conditions $y(0) = y(1) = 0$, and subject to the constraint $\int_0^1 y^2\, dx = 2$.