Math 583 B
Homework Assignment # 4

Due February 18, 1999

Integral equations – Fredholm alternative

1. Show that if $K(x,\xi)$ is continuous, then the Fredholm operator

$$A \cdot = \int_a^b K(x,\xi) \ \cdot \ d\xi, \qquad a,b < \infty$$

is bounded.

2. Show that $Tf = f(0)$ is not a bounded linear functional on the space of continuous functions measured with the L^2 norm, but is a bounded linear functional if measured using the uniform norm

$$||f|| = \max_{x\in[a,b]} |f(x)|.$$

3. When does the following integral equation has a solution ?

$$u(x) = f(x) + \lambda \int_0^1 x \ t \ u(t) \ dt.$$

4. (a) Show that if Φ is a linear functional defined on a Hilbert space S, then $\{u \in S | < u, v >= 0$ for every $v \in Ker(\Phi)\}$ is at most one-dimensional.

   (b) Show that if $Ker(\Phi)$ is closed, then Φ is bounded.