Math 583 B
Homework Assignment # 2

Due February 4, 1999

1 Laplace transforms

1. Give an expression for the Laplace transform of $\int_0^t f(\eta)\, d\eta$ in terms of the Laplace transform of f.
2. Give an expression for the Laplace transform of $f'(x)$ in terms of f and of its Laplace transform.

2 Other Transforms

1. Give an expression for the Mellin transform of $xf'(x)$ in terms of the Mellin transform of f.
2. Give an expression for the Mellin transform of $\int_0^x f(t)\, dt$ in terms of the Mellin transform of f.

3 Transforms and pde's

1. Solve the two-dimensional Laplace equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$, in the region $-\infty < x < \infty$, $y > 0$, by taking the Fourier transform (in x) of this equation. Assume that the solution $u(x, y)$ is such that $v_y(x) = u(x, y)$ is in $L^1(\mathbf{R})$ for all y's, and that $v_x(y) = u(x, y)$ remains bounded for all positive values of y.
2. Use the Mellin transform (in x) to solve the partial differential equation

$$x^2 \frac{\partial^2 u}{\partial x^2} + x \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0$$

in the region $0 \le x < \infty$, $0 < y < 1$, with boundary conditions $u(x, 0) = 0$ and $u(x, 1) = \begin{cases} A & \text{if } 0 \le x \le 1 \\ 0 & \text{if } x > 1 \end{cases}$.