# Math 583 B
# Homework Assignment # 13

## Method of Steepest Descents

1. Find the paths of steepest descent and steepest ascent of $\exp(xt^2)$ near the saddle point $t = 0$.

2. The following questions refer to the example discussed in class.

   (a) Find the saddle points and paths of steepest descent and steepest ascent for
   $$\exp[x(-4t^2 + i(5t - t^3))].$$

   (b) Find the leading order behavior of
   $$I_3 = \int_{c_3} \exp[x(-4t^2 + i(5t - t^3))]\, dt$$
   as $x \to +\infty$, where $C_3$ is the steepest descents path going through $t = i$.

   (c) Show that
   $$I_1 = \int_{c_1} \exp[x(-4t^2 + i(5t - t^3))]\, dt$$
   is subdominant when compared to $I_3$ as $x \to +\infty$. The path $C_1$ is the steepest descent path of $\exp[x(-4t^2 + i(5t - t^3))]$ going through $t = -1$.