Math 583 B
Homework Assignment # 12

Due April 29, 1999

Laplace's Method

1. Find the leading behavior, as $x \to +\infty$, of

   (a) $\displaystyle\int_0^{\pi/2} \sqrt{\sin(t)}\, e^{-x\sin^4(t)}\, dt$

   (b) $\displaystyle\int_0^1 \frac{\sqrt{t(1-t)}}{(t+a)^x}\, dt$, where $a > 0$.

2. Show that

$$\int_0^\infty \frac{t^{x-1}e^{-t}}{t+x}\, dt \sim \frac{1}{2x}\Gamma(x), \qquad \text{as } x \to \infty,$$

   where $\Gamma(x)$ is the gamma function.

Method of Stationary Phase

Find the leading order behavior, as $x \to +\infty$, of

1. $\displaystyle\int_0^1 e^{ixt^2}\cosh(t^2)\, dt$
2. $\displaystyle\int_0^1 \cos(xt^4)\tan(t)\, dt$
3. $\displaystyle\int_{-1}^1 \sin[x(t-\sin(t))]\sinh(t)\, dt.$