Math 583 B
Homework Assignment # 11

Due April 22, 1999

Asymptotic expansions of integrals

1. Find the leading order behavior, as $x \to 0^+$, of

   (a) $\displaystyle\int_x^1 \sin(xt)\ dt$

   (b) $\displaystyle\int_0^1 \frac{e^{-xt}}{1+t^2}\ dt$

2. Find the full asymptotic expansion, as $x \to 0^+$, of $\displaystyle\int_0^1 \frac{e^{-t}}{1+x^2t^3}\ dt$.

3. Find the asymptotic expansion of $I(x) = \displaystyle\int_x^\infty e^{it}\ t^{-1}\ dt$ as $x \to +\infty$. Show that

$$\begin{aligned}\int_x^\infty \cos(t)\ t^{-1}\ dt \ \sim\ & \left(\frac{1}{x^2} - \frac{3!}{x^4} + \frac{5!}{x^6} + \ldots\right)\cos(x) \\ & - \left(\frac{1}{x} - \frac{2!}{x^3} + \frac{4!}{x^5} + \ldots\right)\sin(x)\end{aligned}$$

   as $x \to +\infty$.