Math 583 B
Homework Assignment # 10

Due April 15, 1999

Perturbation theory for eigenvalue problems

1. Consider the family of operators

$$A_\epsilon = -\frac{d^2}{dx^2} + \epsilon \sin(x),$$

acting on $L^2([0,\pi])$, with boundary conditions $u(0) = u(\pi) = 0$.

   (a) Find the eigenvalues and eigenfunctions of this operator when $\epsilon = 0$.

   (b) Find the first order correction to the smallest eigenvalue of A_ϵ.

2. Using perturbation theory, estimate the eigenvalues and eigenvectors of the matrices

   (a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1-\epsilon & 0 & 3 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 1-\epsilon \\ \epsilon & 1 \end{pmatrix}$

Poincaré-Lindstedt method for nonlinear oscillators

1. Consider the equation

$$\ddot{u} + u + \epsilon u^5 = 0, \qquad t > 0, \quad 0 < \epsilon << 1,$$

with boundary conditions $u(0) = 1$ and $\dot{u}(0) = 0$.

   (a) Develop a regular perturbation expansion and find the solution at order ϵ. Show that a secular term is present.

   (b) Apply the Poincaré-Lindstedt method to this problem and find how the frequency should be "renormalized" in order to remove the secular term.

   (c) In qualitative terms, what happens for the problem

$$\ddot{u} + u + \epsilon u^4 = 0, \qquad t > 0, \quad 0 < \epsilon << 1, \quad u(0) = 1, \quad \dot{u}(0) = 0 \ ?$$