Math 583 B
Homework Assignment # 1

Due January 28, 1999

1 Fourier transforms

1. Obtain the Fourier transforms of the following functions
   (a) $e^{-|x|}$
   (b) $\dfrac{1}{x^2+a^2}$, where $a^2 > 0$.
   (c) $\dfrac{1}{(x^2+a^2)^2}$, where $a^2 > 0$.
2. Use (a) and (b) of Problem 1 to find the inverse Fourier transform of the Fourier transform of $\dfrac{1}{x^2+a^2}$, where $a^2 > 0$. Are you surprised by the result of this calculation ? Explain.

2 Laplace Transforms

1. Evaluate the inverse Laplace transform of $\mathcal{L}(s) = \dfrac{1}{s^2+w^2}$, where w is a real constant.
2. In this problem, we consider the inverse Laplace transform of s^{-a}, where $0 < a < 1$.
   (a) Evaluate the inverse Laplace transform of s^{-a} $(0 < a < 1)$, were the branch cut is taken along the negative real axis.
   (b) Express the result of part (a) in terms of the Gamma function
   $$\Gamma(z) = \int_0^\infty u^{z-1}\, e^{-u}\, du,$$
   defined for $\Re e(z) > 0$.
   (c) Show that $\Gamma(n+1) = n\Gamma(n)$ and evaluate $\Gamma(n)$ when n is a positive integer.