Math 583 A
A short DSTOOL primer

1 Startup

DSTOOL can be run on any of the linux machines in the undergraduate computer room, MATH 226-228. Below are the steps to follow to start DSTOOL and define a dynamical system using the parser utility.

1. Login to a linux machine, and start X-windows by typing **startx**.

2. Type **dstool_tk**. The following window appears:

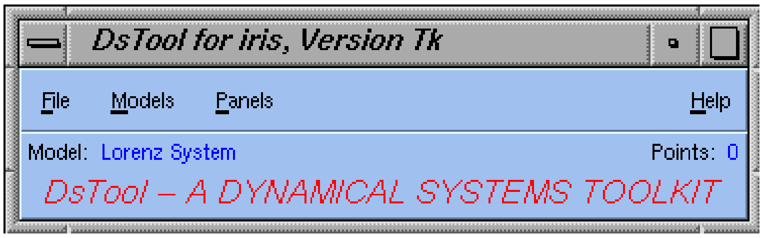

Figure 1: Main DSTOOL window.

3. In the **Models** menu, select **Parser...**. The window shown in Figure 2 pops up. This is where you define the dynamical system you want to study. For instance, for the equation $u_{xx} - u + u^3 - \gamma u^5 = 0$, type

$$x' = y$$
$$y' = x - x\,\hat{}\,3 + \text{gamma}\ x\,\hat{}\,5$$

INITIAL gamma 0.2

RANGE x -3 3 y -3 3

Then, click on the button **Build model**. If you do not see any error message, DSTOOL is ready to run.

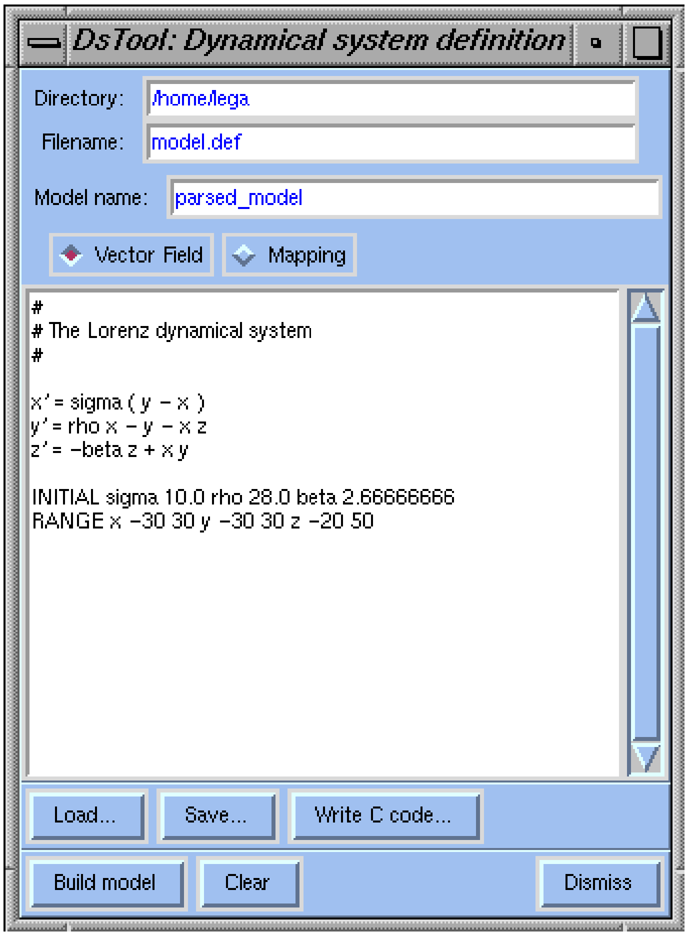

Figure 2: DSTOOL Parser window.

2 Drawing trajectories

In the **Panels** menu of the main window (see Figure 1), select **Two-D...**. The window, called "2-D View" and shown in Figure 3, pops up. Click in

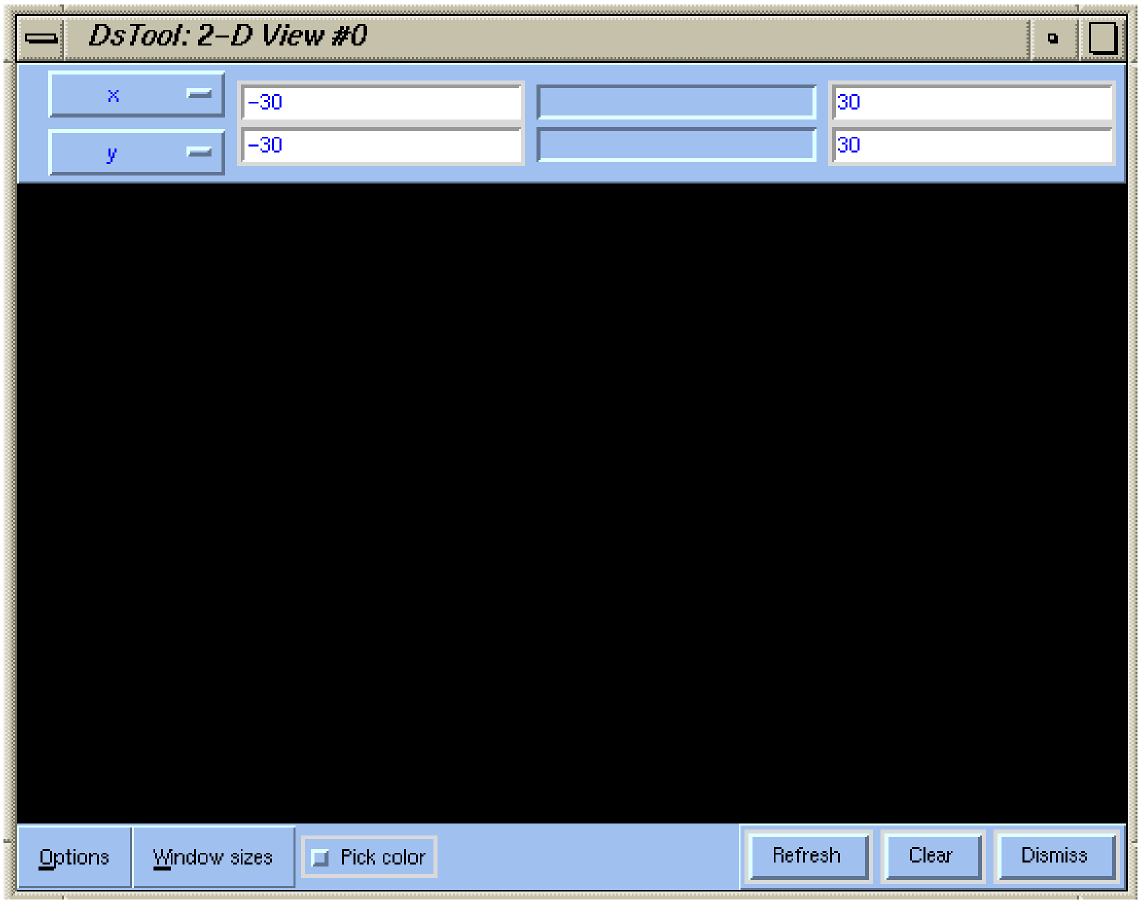

Figure 3: DSTOOL "2-D View" window.

the black region with the left mouse button to select an initial condition and click on the middle button once to draw the corresponding trajectory. To iterate backwards in time, click the right button once and the middle button again.

If you do not like using the mouse, go back to the Main window (Figure 1) and select **Orbits** in the **Panels** menu. The window shown in Figure 4 appears, in which you can select the direction of iteration (**Forwards** or **Backwards**), the initial condition, and the numerical scheme used to integrate the dynamical system (click on **Propagation...**). You can also decide to continue drawing a given trajectory by clicking on the **Continue** button.

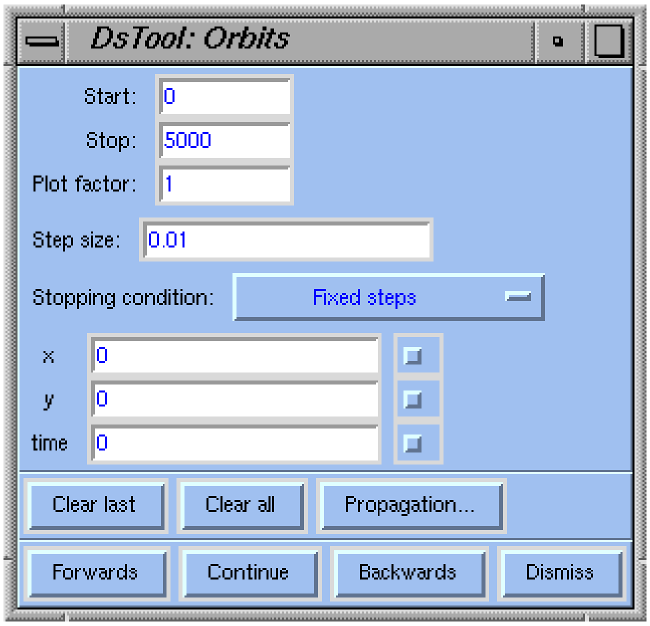

Figure 4: DSTOOL "Orbits" window.

3 Fixed points and stable and unstable manifolds

In the **Panels** menu of the main window (Figure 1), select **Fixed Points...**; the window shown in Figure 5 appears. Click on the **Find fixed points** button a few times to make sure DSTOOL finds all of the fixed points. For each fixed point found, the following information will appear on the shell window where you started DSTOOL from:

NEW fixed point at: 1.3135596251120582e-10 -1e-10 0
Eigenvalues: 0.99999999999949984+0I -0.99999999999949996+0I
Type: SADDLE

and the correponding fixed points will also be marked in the phase plane of "2-D View". You can compute the stable (in blue) and unstable (in red) manifolds by clicking on **Compute 1-d manifolds**. If the manifolds are not visible or do not have enough points, change the value of **stable manifold steps** and **unstable manifold steps** in the "Fixed Points" window (Figure 5). You can also change the algorithm used to find the fixed points, which is either Newton's method or the secant method. The "Seed algorithm" is used to find the initial condition for the above methods. If you use "Selected Point", you have to select the initial condition by clicking in the "2-D View" window.

Draw the stable and unstable manifolds for the dynamical system $u_{xx} - u + u^3 - \gamma u^5 = 0$ and draw a few trajectories. Change the value of γ. Could

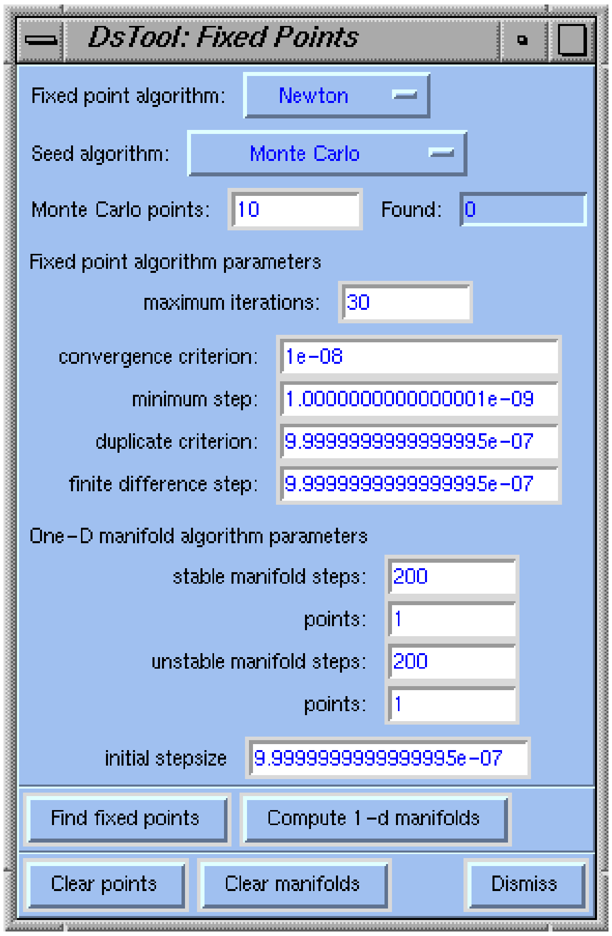

Figure 5: DSTOOL "Fixed Points" window.

you have predicted the form of the phase portraits by looking at the form of the associated potential ?

To print out a copy of the two-dimensional phase portrait, click on **Options** at the bottom of the "2-D View" window and select **Snapshot...**. Save the image in a file (called for instance *twod.snap*) and then type **xv twod.snap** from a shell window to visualize the file. Once you have started xv, click on the image with the right mouse button to make the xv command window appear. Then click on the **print** button of the xv command window to print the image.

Notes:

1. If you start xv and the image appears white, take another snapshot of the Two-D view in DSTOOL using a **PPM**, **GIF**, or **TIFF** format, instead of using the **BITMAP** format, and proceed as above.

2. Do not use the print command from DSTOOL because it creates huge postscript files which take forever to print.

4 Geomview

Geomview allows you to visualize trajectories in three-dimensional phase spaces. Below is an example for the Lorenz system, defined in Figure 2. In order to tell DSTOOL you want to study the Lorenz system, select **Vector Fields** and then **Lorenz System** in the **Models** menu of the main window (Figure 1). Draw a single trajectory in the two-D window, as shown in Figure 6. Now select **Three-D (Geomview)...** in the **Panels** menu of the main window (Figure 1), and click on **Send to geomview** in the window which then pops up (not shown here). The computer will start Geomview and the trajectory computed by DSTOOL will be drawn in the three dimensional phase space of the Lorenz system (as shown in Figure 7). You can rotate, translate or scale this three-D view by using the corresponding buttons of the Geomview menu.

To print from Geomview, select **Save** in the **File** menu and save the figure in a postscript file (say geom.ps). Then, print the file, i.e. execute the command **lpr geom.ps** from a shell window.

5 Quiting DSTOOL

When you are done, quit DSTOOL (use the **Exit** command in the **File** menu of the main window), and then quit X-windows. To do so, click with the left mouse button in the background area of the desktop, select **Exit** and confirm. Then log yourself off the computer. If the **Exit** command in X-windows does not work, type **Ctrl Alt Backspace** to quit X-windows.

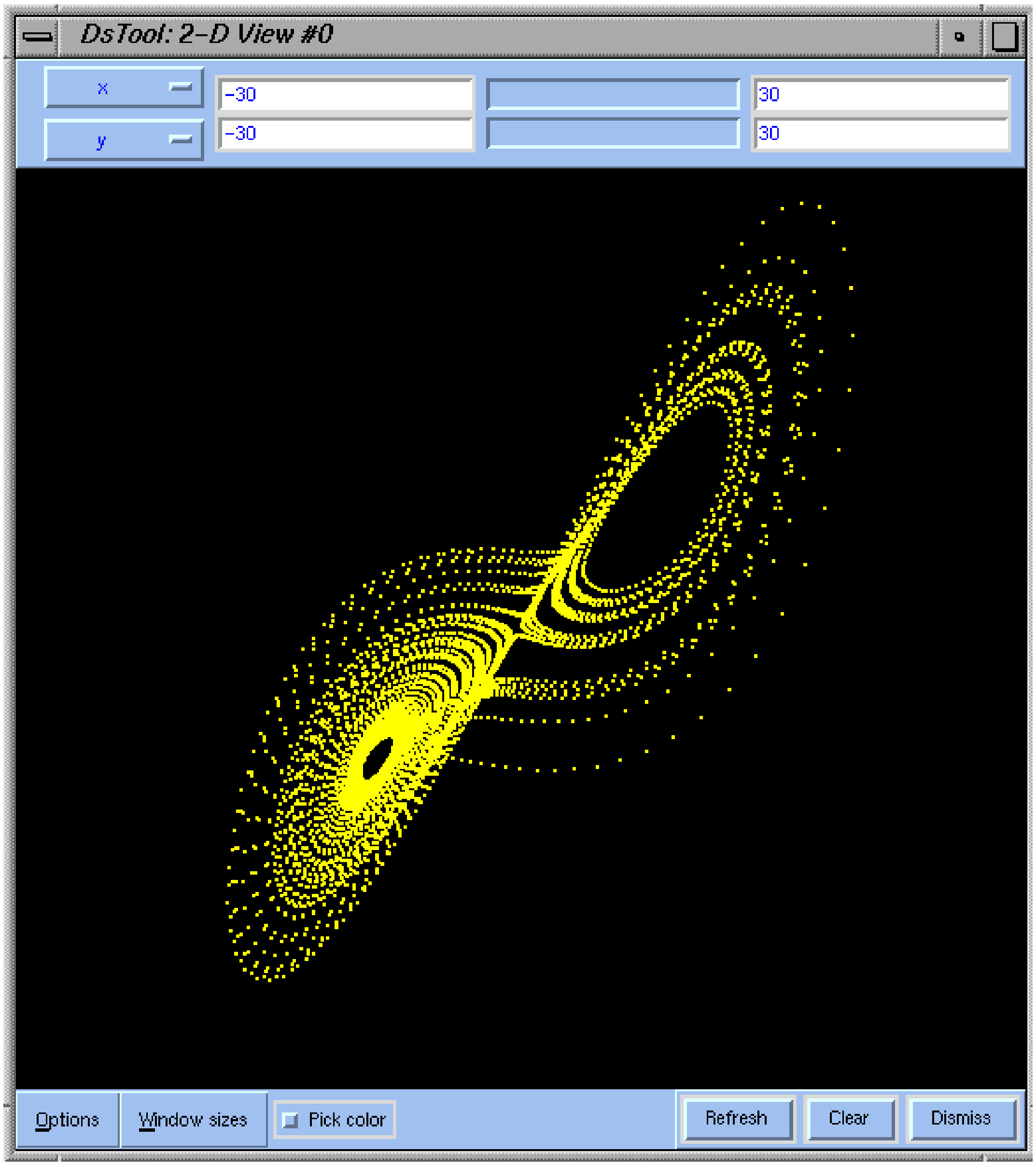

Figure 6: Two-dimensional projection of a trajectory of the Lorenz system.

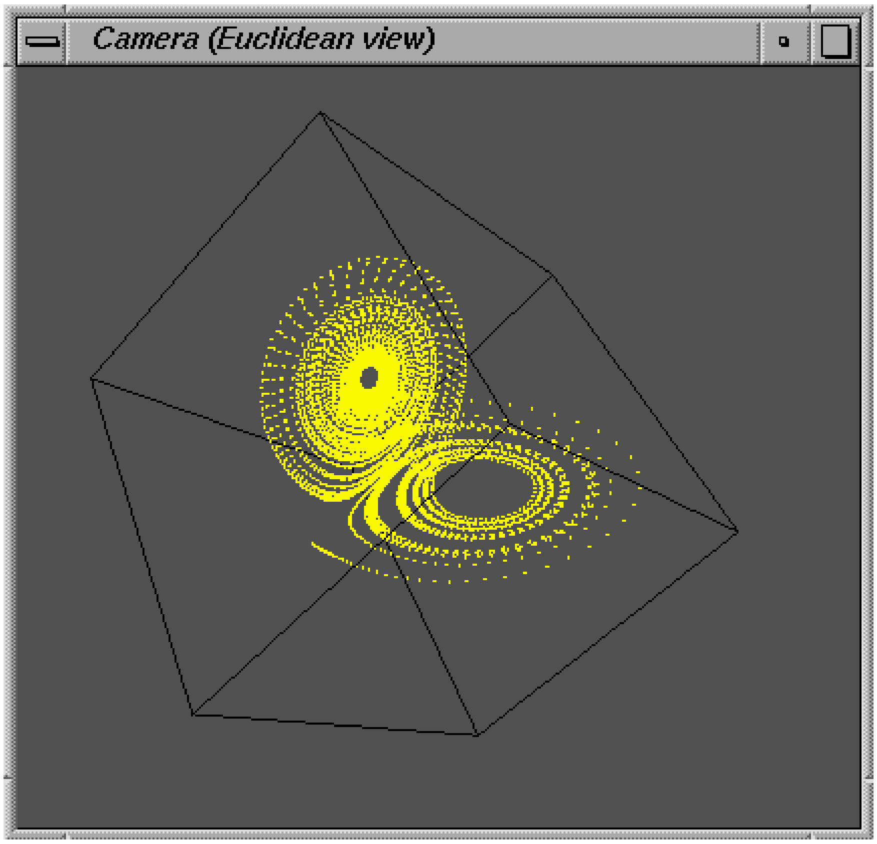

Figure 7: Trajectory of the Lorenz system as seen in Geomview.

6 Selected problems

Below is a short list of dynamical systems, which you can analyze by hand (i.e. find the fixed points, perform their linear stability analysis and/or find potential functions if any), and with DSTOOL. Have fun !

1. The Van der Pol oscillator: $\ddot{x} + \omega^2 x + \epsilon(x^2 - 1)\dot{x} = 0$.
2. $\dot{x} = x(2 - y - x)$, $\dot{y} = y(4x - x^2 - 3)$.
3. $\dot{r} = r(3 - r - 2s)$, $\dot{s} = s(2 - r - s)$.
4. $u_{xx} - u + u^3 - \gamma u^5 = 0$, for positive values of γ.