Homework Assignment # 7

Due October 22, 1998

Contour integration

1. Let $f(z)$ be a meromorphic function defined inside and on a simple closed contour C, with no zeroes or poles on C. Show that
$$I = \frac{1}{2\pi i}\int_C \frac{f'(z)}{f(z)}\,dz = N - P = \frac{1}{2\pi}\left[\arg(f(z))\right]_C,$$
where N and P are the number of zeroes and poles, respectively, of f inside C, $\arg(f(z))$ is defined by $f(z) = |f(z)|\exp\left[i\arg(f(z))\right]$, and $\left[\arg(f(z))\right]_C$ is the change of $\arg(f(z))$ over C.

   This result is called the **Argument Principle**.

2. Use a rectangular contour with corners at $\pm R$ and $\pm R + i\pi/k$ and appropriate indentation to find $\displaystyle\int_0^\infty \frac{x}{\sinh(kx)}\,dx$.

3. Find $\displaystyle\int_0^\infty \frac{\sin^2(x)}{x^2}\,dx$.

   Hint: Use principal value integrals to show that
$$\int_0^\infty \frac{\cos(kx)-\cos(mx)}{x^2}\,dx = -\frac{\pi}{2}(|k|-|m|)$$
and let $k = 2$ and $m = 0$.

4. Keener's book: # 6.4.7 page 280. Assume a and x are real.

5. Keener's book: # 6.4.8 page 280.

6. Keener's book: # 6.4.9 page 280.