Math 583 A
Homework Assignment # 6

Due October 15, 1998

1 Multivalued functions - Poles and residues

1. Determine all possible values and give the principal value of the following numbers (put in the form $x + iy$):

$$\text{(a)}\, i^{1/2} \qquad \text{(b)}\, \ln(1+i\sqrt{3}) \qquad \text{(c)}\, \ln(i^3) \qquad \text{(d)}\, \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

2. Discuss the type (removable, pole, essential, branch point) of all singular points in the finite complex plane of the following functions. If the type is pole, specify the order of the pole.

$$\text{(a)}\ \frac{e^{z^2}-1}{z^2} \qquad \text{(b)}\ \frac{e^{2z}-1}{z^2} \qquad \text{(c)}\ e^{\tan(z)} \qquad \text{(d)}\ \frac{z^3}{z^2+z+1}$$

$$\text{(e)}\ \frac{z^{1/3}-1}{z-1} \qquad \text{(f)}\ \ln(1+z^{1/2}) \qquad \text{(g)}\ \operatorname{sech}(z)$$

2 Cauchy's residue theorem

1. Evaluate the integrals $\frac{1}{2\pi i}\int_C f(z)\,dz$, where C is the circle of radius 1 centered at the origin and $f(z)$ is given below:

$$\text{(a)}\ \frac{z+1}{2z^3-3z^2-2z} \qquad \text{(b)}\ \frac{\cosh(1/z)}{z} \qquad \text{(c)}\ \frac{e^{-\cosh(z)}}{4z^2+\pi^2}$$

$$\text{(d)}\ \frac{\ln(z+2)}{2z+1}, \text{ principal branch} \qquad \text{(e)}\ \frac{z+1/z}{z(2z-1/(2z))}$$

2. Keener's book: # 6.4.1 page 279.

3. Keener's book: # 6.4.2 page 279.