Problems on dimensional analysis and scalings
MCB / MATH 303

1. Consider the following differential equation,

$$\frac{dx}{dt} = \lambda x - \gamma x^3,$$

where λ and γ are both non-zero and have the same sign.

1. What is the dimension of λ?
2. What is the dimension of γ? Your answer should be in terms of the dimension of x, denoted by $[x]$.
3. Let t_0 be a characteristic time scale for this problem. Define a dimensionless time variable $\tau = t/t_0$, and show that you can make a change of variable from t to τ, to "get rid" of the parameter λ.
4. Can you find a change of variable that would allow you to "get rid" of γ as well?

2. Consider the following model

$$\frac{\partial u}{\partial t} = \mu u + \alpha \frac{\partial^2 u}{\partial x^2} - \beta u^3, \qquad u \in \mathbb{R}, \quad \mu, \alpha, \beta > 0.$$

1. What are the dimensions of the parameters α, β and μ? Write $[u]$ for the dimension of u.
2. How many relevant parameters does this model have? Explain.

3. Write the model below in dimensionless form by defining $\tau = \lambda k b t/\delta^2$ and appropriate variables x, y and v.

$$\begin{aligned} \frac{dX}{dt} &= \lambda - \delta X - bVX \\ \frac{dY}{dt} &= bVX - aY \\ \frac{dV}{dt} &= kY - \kappa V \end{aligned}$$